

IOWA STATE UNIVERSITY

ECpE Department

OpenDSS – Modeling A Transformer

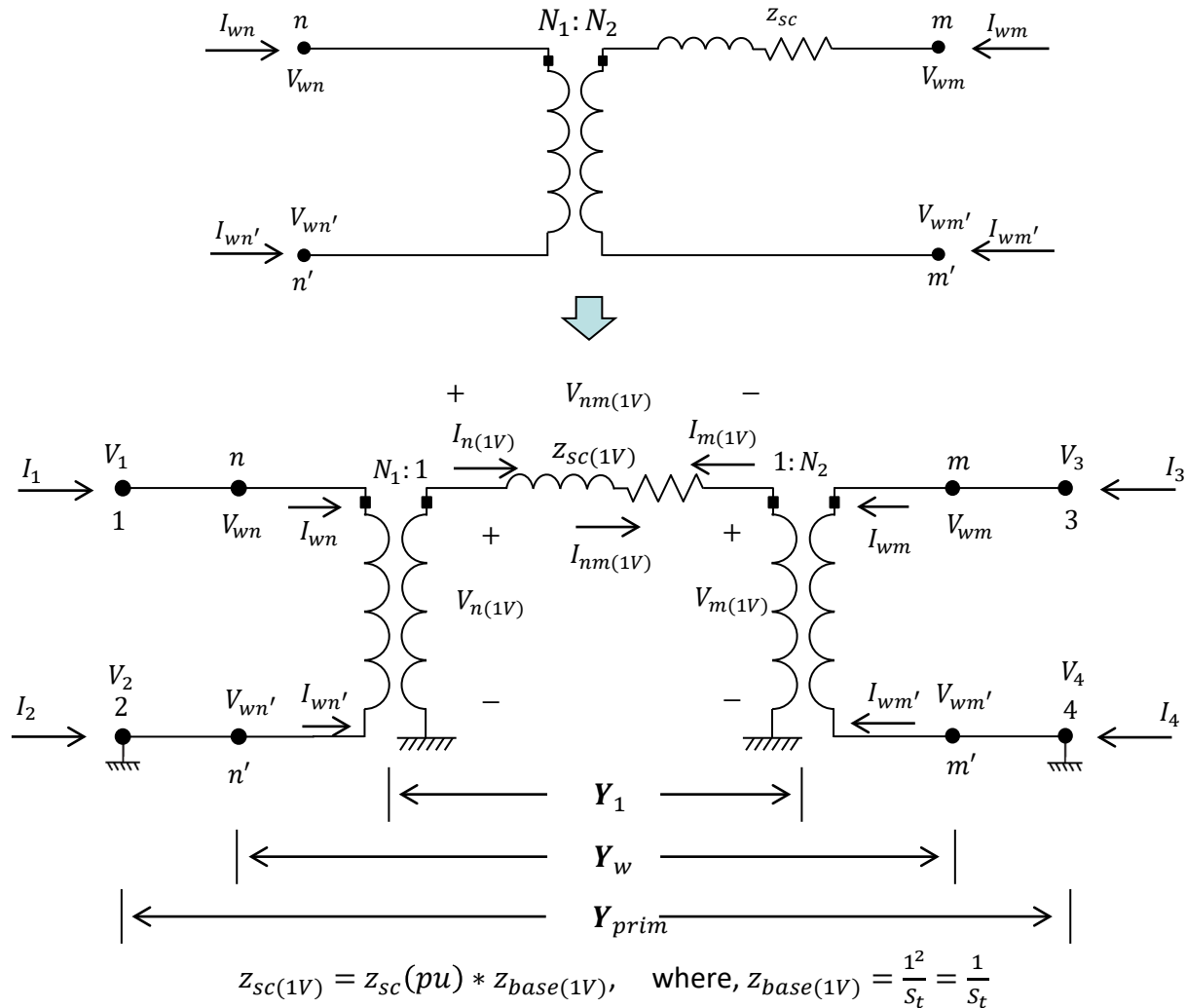
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Modeling A Transformer in OpenDSS

1. Single-phase Transformer
2. Three-phase Delta-Grounded Wye Step-down Transformer
3. Three-phase Delta-Grounded Wye Step-up Transformer
4. Three-phase Ungrounded Wye-Delta Step-down Transformer
5. Three-phase Ungrounded Wye-Delta Step-up Transformer
6. Three-phase Grounded Wye-Delta Step-down Transformer
7. Three-phase Grounded Wye-Grounded Wye Transformer
8. Three-phase Delta-Delta Transformer

Modeling A Transformer in OpenDSS

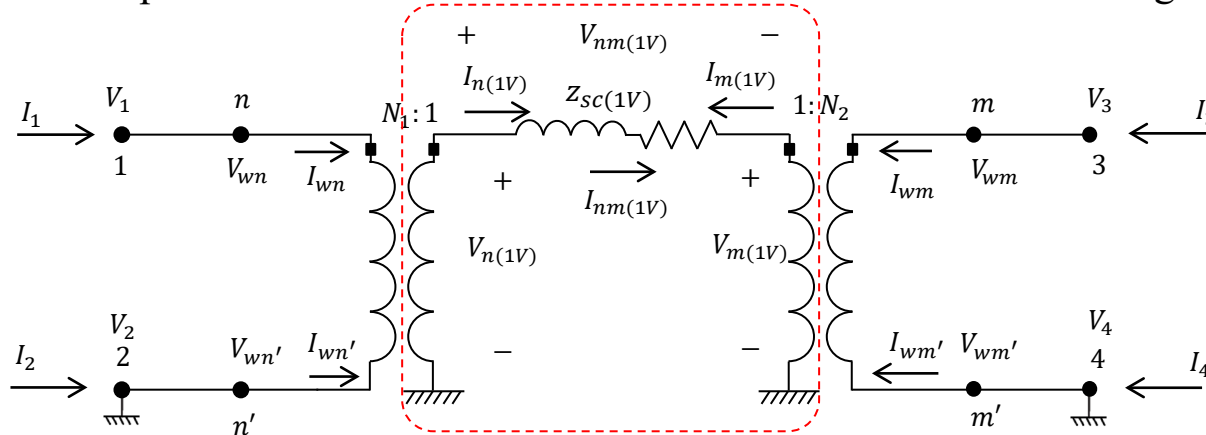
1. Single-phase Transformer



Modeling A Transformer in OpenDSS

1. Single-phase Transformer

Step 1: Compute the nodal admittance matrix on a one turn or one voltage base.



$$\begin{cases} I_n(1V) = I_{nm}(1V) \\ I_m(1V) = -I_{nm}(1V) \end{cases} \Rightarrow \begin{bmatrix} I_n(1V) \\ I_m(1V) \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} I_{nm}(1V) \Rightarrow \mathbf{I}_1 = \mathbf{B} I_{nm}(1V), \text{ where } \mathbf{I}_1 = \begin{bmatrix} I_n(1V) \\ I_m(1V) \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$

$$V_{nm}(1V) = V_n(1V) - V_m(1V) \Rightarrow V_{nm}(1V) = [1 \quad -1] \begin{bmatrix} V_n(1V) \\ V_m(1V) \end{bmatrix} \Rightarrow V_{nm}(1V) = \mathbf{B}^T \mathbf{V}_1, \text{ where } \mathbf{V}_1 = \begin{bmatrix} V_n(1V) \\ V_m(1V) \end{bmatrix}$$

$$\text{Also, } V_{nm}(1V) = Z_{sc}(1V) I_{nm}(1V), \text{ i.e., } I_{nm}(1V) = y_{sc}(1V) V_{nm}(1V)$$

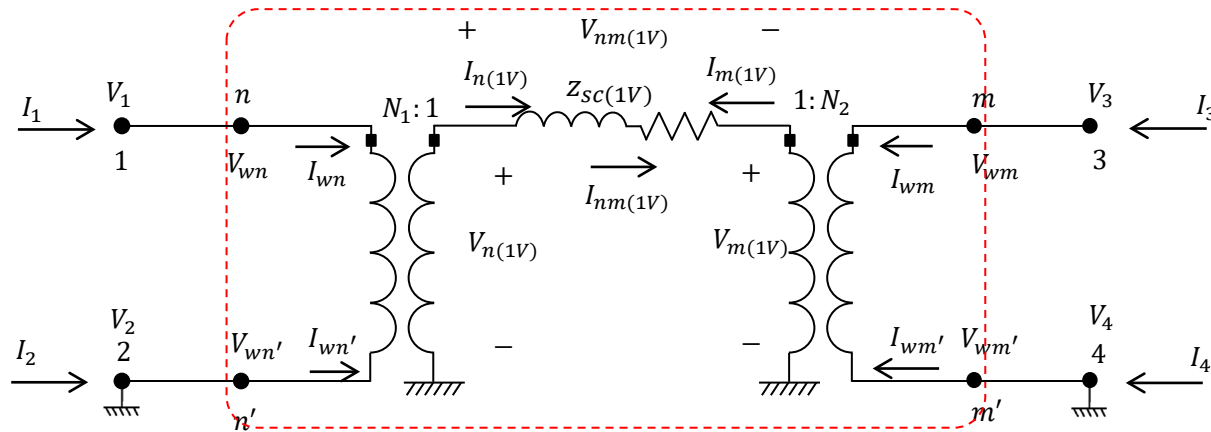
Let $\mathbf{I}_1 = \mathbf{Y}_1 \mathbf{V}_1$, where, $\mathbf{Y}_1$ is the admittance matrix on a one turn or one voltage base. What is $\mathbf{Y}_1$?

$$\left. \begin{aligned} \mathbf{I}_1 &= \mathbf{Y}_1 \mathbf{V}_1 \\ \mathbf{I}_1 &= \mathbf{B} I_{nm}(1V) \end{aligned} \right\} \Rightarrow \left. \begin{aligned} \mathbf{B} I_{nm}(1V) &= \mathbf{Y}_1 \mathbf{V}_1 \\ V_{nm}(1V) &= \mathbf{B}^T \mathbf{V}_1 \end{aligned} \right\} \Rightarrow \left. \begin{aligned} \mathbf{B} I_{nm}(1V) &= \mathbf{Y}_1 (\mathbf{B}^T)^{-1} V_{nm}(1V) \\ I_{nm}(1V) &= y_{sc}(1V) V_{nm}(1V) \end{aligned} \right\} \Rightarrow \mathbf{B} y_{sc}(1V) V_{nm}(1V) = \mathbf{Y}_1 (\mathbf{B}^T)^{-1} V_{nm}(1V) \Rightarrow \mathbf{Y}_1 = \mathbf{B} y_{sc}(1V) \mathbf{B}^T$$

Modeling A Transformer in OpenDSS

1. Single-phase Transformer

Step 2: Consider winding turns ratio.



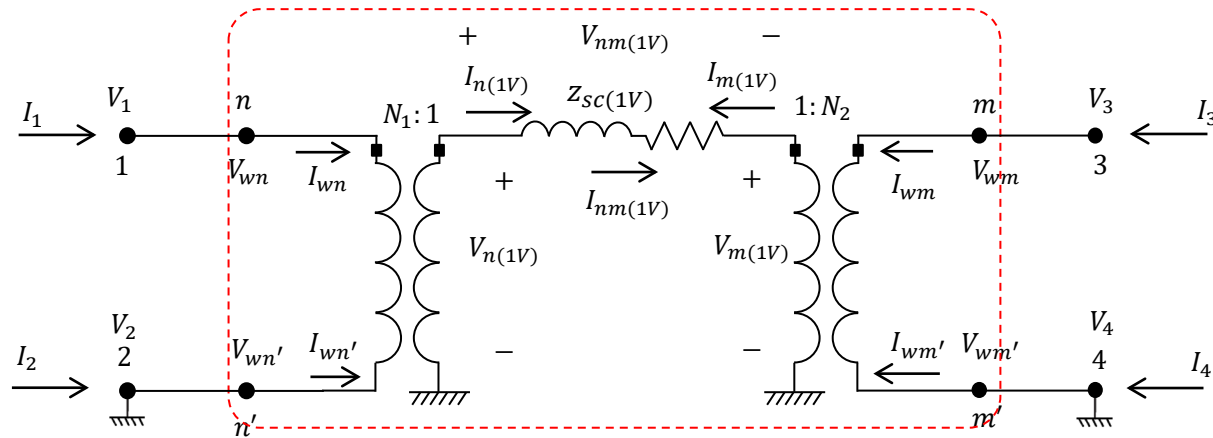
$$\begin{cases} I_{wn} = I_{n(1V)}/N_1 \\ I_{wn'} = -I_{n(1V)}/N_1 \\ I_{wm} = I_{m(1V)}/N_2 \\ I_{wm'} = -I_{m(1V)}/N_2 \end{cases} \Rightarrow \begin{bmatrix} I_{wn} \\ I_{wn'} \\ I_{wm} \\ I_{wm'} \end{bmatrix} = \begin{bmatrix} 1/N_1 & 0 \\ -1/N_1 & 0 \\ 0 & 1/N_2 \\ 0 & -1/N_2 \end{bmatrix} \begin{bmatrix} I_{n(1V)} \\ I_{m(1V)} \end{bmatrix} \Rightarrow \mathbf{I}_w = \mathbf{N} \mathbf{I}_1, \text{ where } \mathbf{I}_w = \begin{bmatrix} I_{wn} \\ I_{wn'} \\ I_{wm} \\ I_{wm'} \end{bmatrix}, \mathbf{N} = \begin{bmatrix} 1/N_1 & 0 \\ -1/N_1 & 0 \\ 0 & 1/N_2 \\ 0 & -1/N_2 \end{bmatrix}$$

$$\begin{cases} V_{wn} - V_{wn'} = V_{n(1V)}N_1 \\ V_{wm} - V_{wm'} = V_{m(1V)}N_2 \end{cases} \Rightarrow \begin{bmatrix} V_{n(1V)} \\ V_{m(1V)} \end{bmatrix} = \begin{bmatrix} 1/N_1 & -1/N_1 & 0 & 0 \\ 0 & 0 & 1/N_2 & -1/N_2 \end{bmatrix} \begin{bmatrix} V_{wn} \\ V_{wn'} \\ V_{wm} \\ V_{wm'} \end{bmatrix} \Rightarrow \mathbf{V}_1 = \mathbf{N}^T \mathbf{V}_w, \text{ where } \mathbf{V}_w = \begin{bmatrix} V_{wn} \\ V_{wn'} \\ V_{wm} \\ V_{wm'} \end{bmatrix}$$

Modeling A Transformer in OpenDSS

1. Single-phase Transformer

Step 2: Consider winding turns ratio.



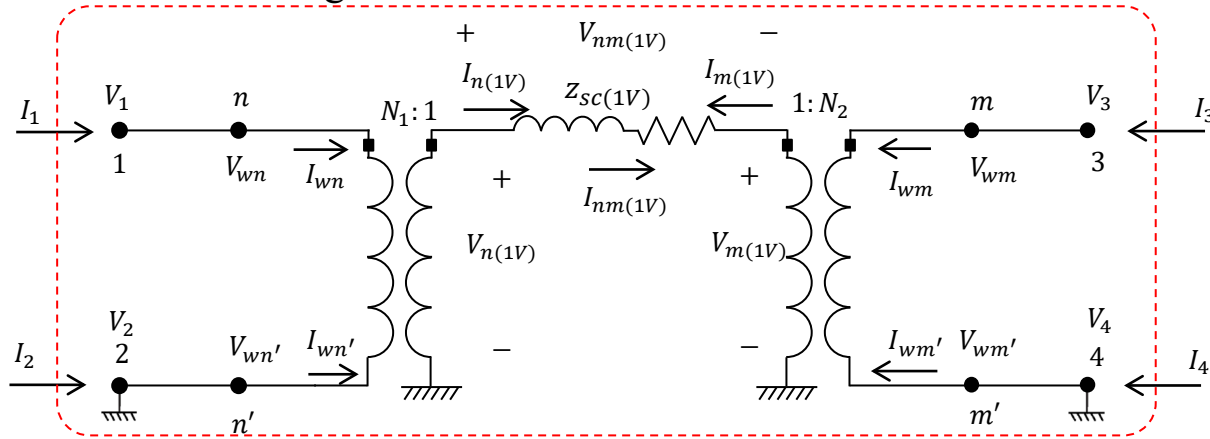
Next, let $I_w = Y_w V_w$. How to obtain Y_w ?

$$\left. \begin{aligned} I_w &= Y_w V_w \\ I_w &= N I_1 \\ V_1 &= N^T V_w \end{aligned} \right\} \Rightarrow \left. \begin{aligned} N I_1 &= Y_w (N^T)^{-1} V_1 \\ I_1 &= Y_1 V_1 \end{aligned} \right\} \Rightarrow N Y_1 V_1 = Y_w (N^T)^{-1} V_1 \Rightarrow \left. \begin{aligned} Y_w &= N Y_1 N^T \\ Y_1 &= B y_{sc(1V)} B^T \end{aligned} \right\} \Rightarrow Y_w = N B y_{sc(1V)} B^T N^T$$

Modeling A Transformer in OpenDSS

1. Single-phase Transformer

Step 3: Consider winding connections.



Let $\mathbf{I}_{prim} = \mathbf{Y}_{prim} \mathbf{V}_{prim}$. How to obtain $\mathbf{Y}_{prim}$?

$$\left. \begin{array}{l} I_1 = I_{wn} \\ I_2 = I_{wn'} \\ I_3 = I_{wm} \\ I_4 = I_{wm'} \end{array} \right\} \Rightarrow \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_{wn} \\ I_{wn'} \\ I_{wm} \\ I_{wm'} \end{bmatrix} \Rightarrow \mathbf{I}_{prim} = \mathbf{A} \mathbf{I}_w, \text{ where, } \mathbf{I}_{prim} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}, \mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

$$\left. \begin{array}{l} V_1 = V_{wn} \\ V_2 = V_{wn'} \\ V_3 = V_{wm} \\ V_4 = V_{wm'} \end{array} \right\} \Rightarrow \begin{bmatrix} V_{wn} \\ V_{wn'} \\ V_{wm} \\ V_{wm'} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} \Rightarrow \mathbf{V}_w = \mathbf{A}^T \mathbf{V}_{prim}, \text{ where, } \mathbf{V}_{prim} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}.$$

$$\left. \begin{array}{l} \mathbf{I}_{prim} = \mathbf{Y}_{prim} \mathbf{V}_{prim} \\ \mathbf{I}_{prim} = \mathbf{A} \mathbf{I}_w \end{array} \right\} \Rightarrow \left. \begin{array}{l} \mathbf{A} \mathbf{I}_w = \mathbf{Y}_{prim} \mathbf{V}_{prim} \\ \mathbf{V}_w = \mathbf{A}^T \mathbf{V}_{prim} \end{array} \right\} \Rightarrow \left. \begin{array}{l} \mathbf{I}_w = \mathbf{Y}_w \mathbf{V}_w \\ \mathbf{A} \mathbf{I}_w = \mathbf{Y}_{prim} (\mathbf{A}^T)^{-1} \mathbf{V}_w \end{array} \right\} \Rightarrow \left. \begin{array}{l} \mathbf{A} \mathbf{Y}_w \mathbf{V}_w = \mathbf{Y}_{prim} (\mathbf{A}^T)^{-1} \mathbf{V}_w \\ \mathbf{Y}_w = \mathbf{N} \mathbf{B} \mathbf{y}_{sc(1V)} \mathbf{B}^T \mathbf{N}^T \end{array} \right\} \Rightarrow \mathbf{Y}_{prim} = \mathbf{A} \mathbf{N} \mathbf{B} \mathbf{y}_{sc(1V)} \mathbf{B}^T \mathbf{N}^T \mathbf{A}^T$$

Modeling A Transformer in OpenDSS

1. Single-phase Transformer

An example:

// Define a single-phase transformer.

New Transformer.xfmr phases=1 windings=2 xhl=5

~ wdg=1 bus=n kV=66.39 kva=50 %r=2.5

~ wdg=2 bus=m kV=4.16 kva=50 %r=2.5

Transformer name

Number of
phases

Number of
windings

Percent reactance
high-to-low

Specify which winding
will be edited

Rated voltage

Base kVA
rating

Percent
resistance

Computing Y_{prim} using Matlab:

$$y_{sc(1V)} = \frac{1}{z_{sc(1V)}} = \frac{1}{z_{sc}(pu) * z_{base(1V)}} = \frac{1}{[r(pu) + jx(pu)] * \frac{1^2}{S_t}} = \frac{50000}{(0.05 + j0.05)} = 500000 - j500000 \Omega$$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad N = \begin{bmatrix} 1/66390 & 0 \\ -1/66390 & 0 \\ 0 & 1/4160 \\ 0 & -1/4160 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Calculate $Y_{prim} = ANBy_{sc(1V)} B^T N^T A^T =$

$$\begin{bmatrix} 0.00011343 - j0.00011343 & -0.00011343 + j0.00011343 & -0.00181039 + j0.00181039 & 0.00181039 - j0.00181039 \\ -0.00011343 + j0.00011343 & 0.00011343 - j0.00011343 & 0.00181039 - j0.00181039 & -0.00181039 + j0.00181039 \\ -0.00181039 + j0.00181039 & 0.00181039 - j0.00181039 & 0.02889238 - j0.02889238 & -0.02889238 + j0.02889238 \\ 0.00181039 - j0.00181039 & -0.00181039 + j0.00181039 & -0.02889238 + j0.02889238 & 0.02889238 - j0.02889238 \end{bmatrix}$$

Exported Y_{prim} from OpenDSS:

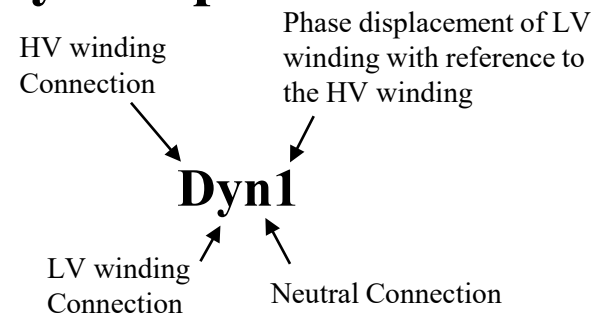
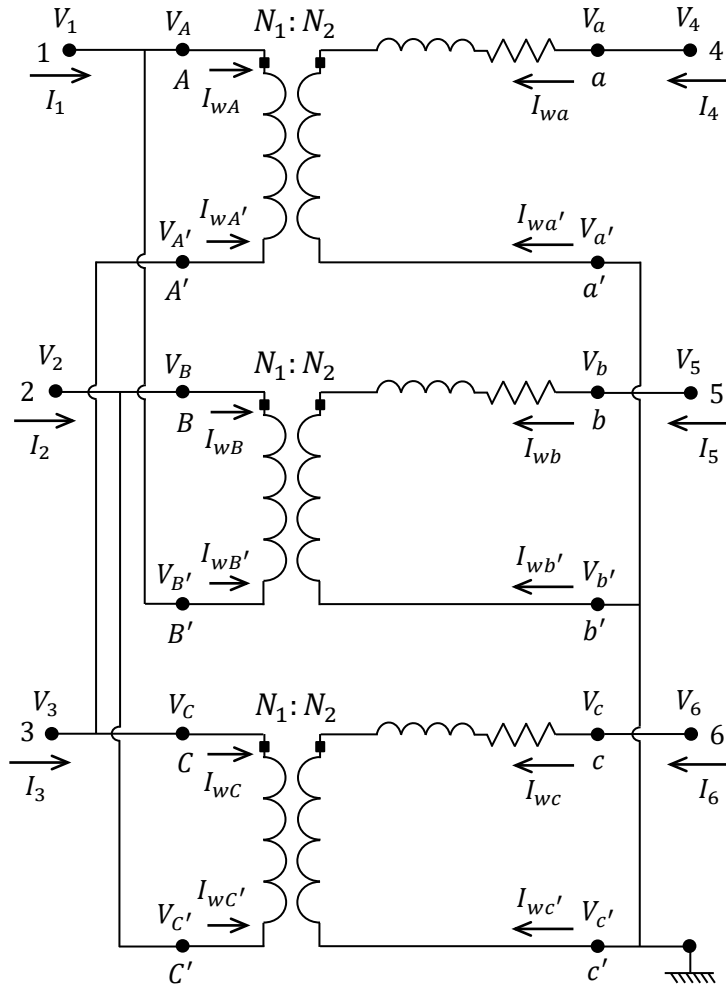
0.000113	-0.00011	-0.00011	0.000113	-0.00181	0.00181	0.00181	-0.00181
-0.00011	0.000113	0.000113	-0.00011	0.00181	-0.00181	-0.00181	0.00181
-0.00181	0.00181	0.00181	-0.00181	0.028892	-0.02889	-0.02889	0.028892
0.00181	-0.00181	-0.00181	0.00181	-0.02889	0.028892	0.028892	-0.02889

G
↓

B
↓

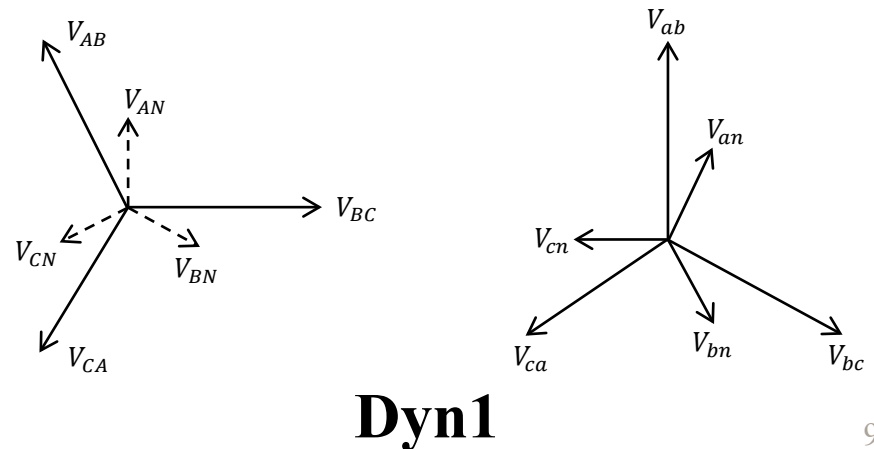
Modeling A Transformer in OpenDSS

2. Three-phase Delta-Grounded Wye Step-down Transformer



Note that

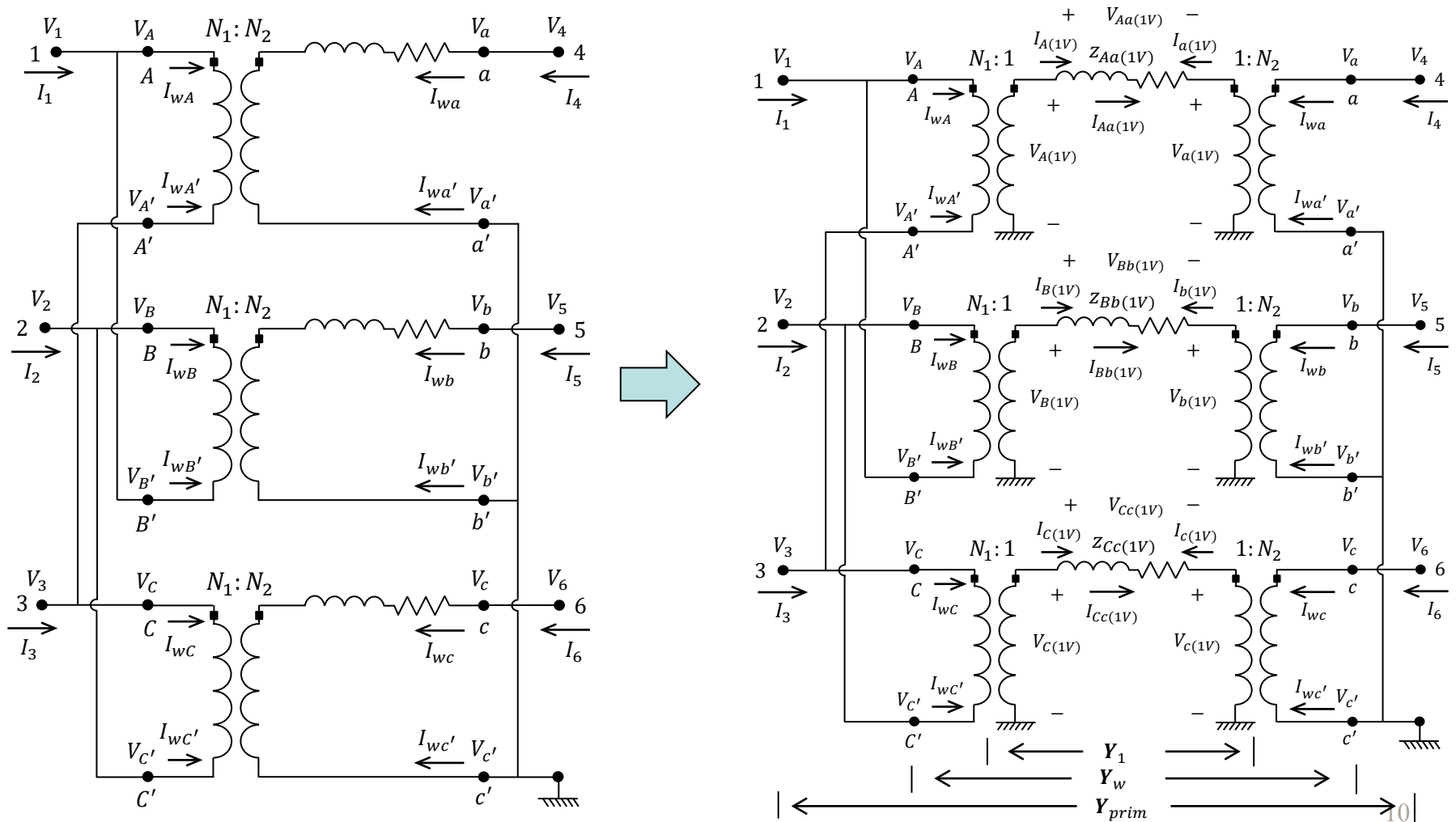
- The HV side always comes before the LV side, regardless of which is the primary winding.
- HV winding is taken as a reference.
- Phase rotation is always counterclockwise.
- 1 = 30°, 2 = 60°, 3 = 90°, 6 = 180° and 12 = 0° or 360°.



https://en.wikipedia.org/wiki/Vector_group

Modeling A Transformer in OpenDSS

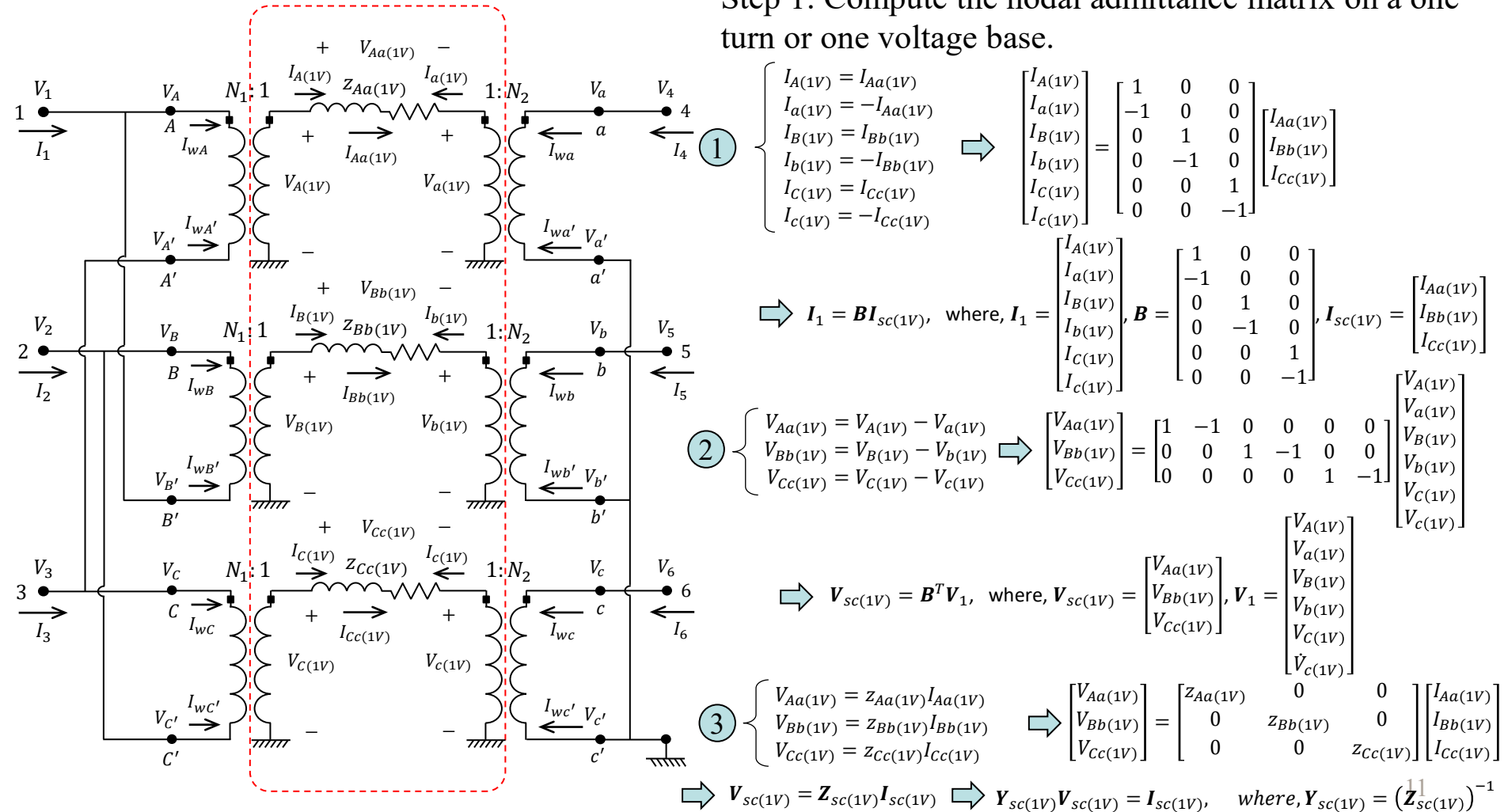
2. Three-phase Delta-Grounded Wye Step-down Transformer



Modeling A Transformer in OpenDSS

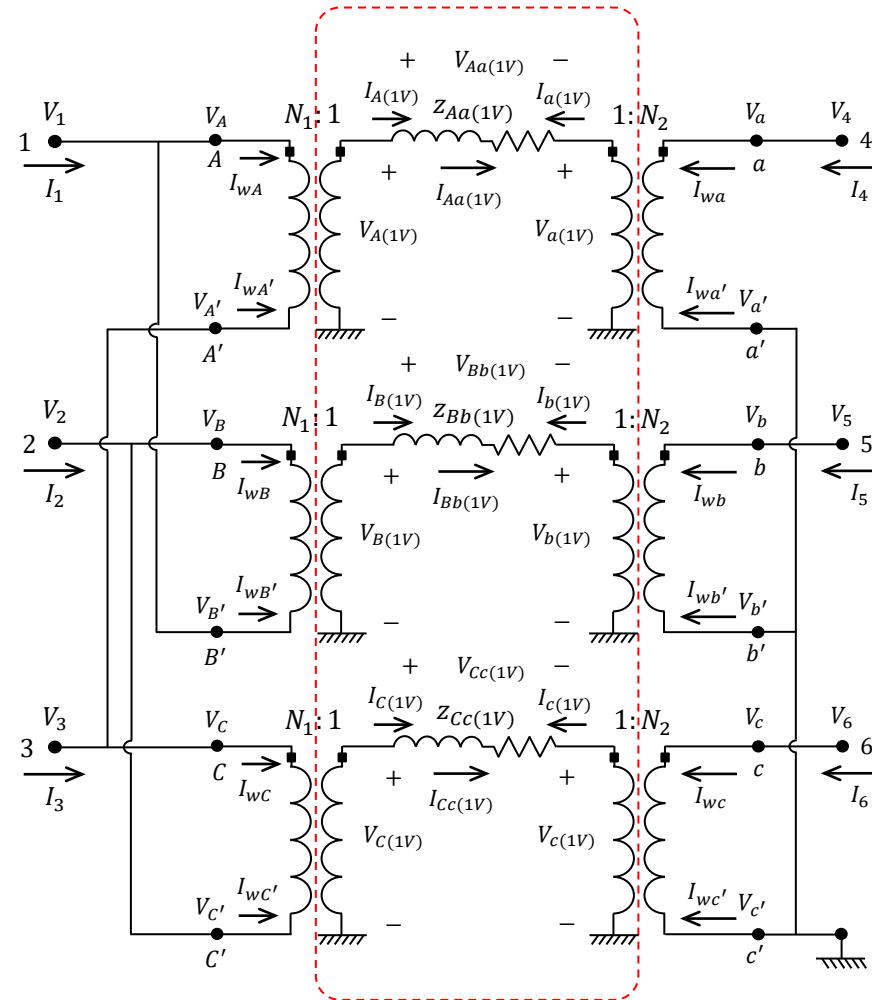
2. Three-phase Delta-Grounded Wye Step-down Transformer

Step 1: Compute the nodal admittance matrix on a one turn or one voltage base.



Modeling A Transformer in OpenDSS

2. Three-phase Delta-Grounded Wye Step-down Transformer



Let $I_1 = Y_1 V_1$, where Y_1 is the admittance matrix on a one turn or one voltage base. What is Y_1 ?

$$\left. \begin{aligned} I_1 &= Y_1 V_1 \\ I_1 &= B I_{sc(1V)} \end{aligned} \right\}$$

$$\Rightarrow \left. \begin{aligned} B I_{sc(1V)} &= Y_1 V_1 \\ V_{sc(1V)} &= B^T V_1 \end{aligned} \right\}$$

$$\Rightarrow \left. \begin{aligned} B I_{sc(1V)} &= Y_1 (B^T)^{-1} V_{sc(1V)} \\ I_{sc(1V)} &= Y_{sc(1V)} V_{sc(1V)} \end{aligned} \right\}$$

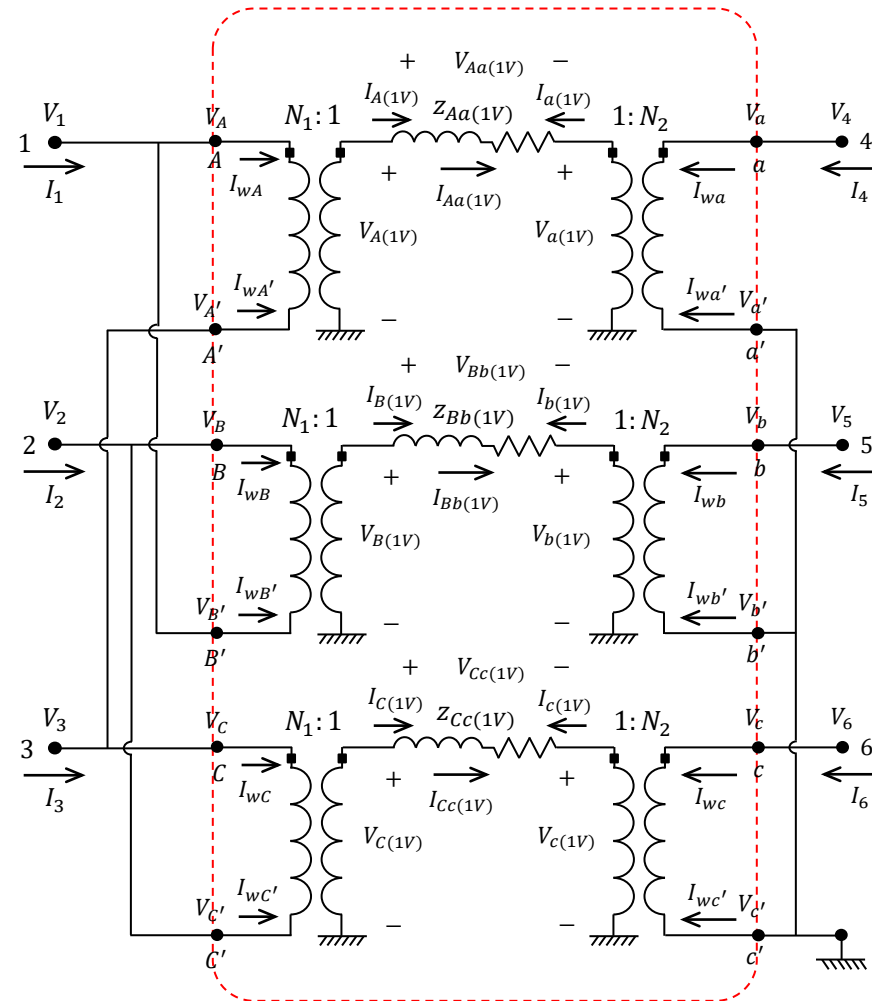
$$\Rightarrow B Y_{sc(1V)} V_{sc(1V)} = Y_1 (B^T)^{-1} V_{sc(1V)}$$

$$\Rightarrow Y_1 = B Y_{sc(1V)} B^T$$

Modeling A Transformer in OpenDSS

2. Three-phase Delta-Grounded Wye Step-down Transformer

Step 2: Consider winding turns ratio.



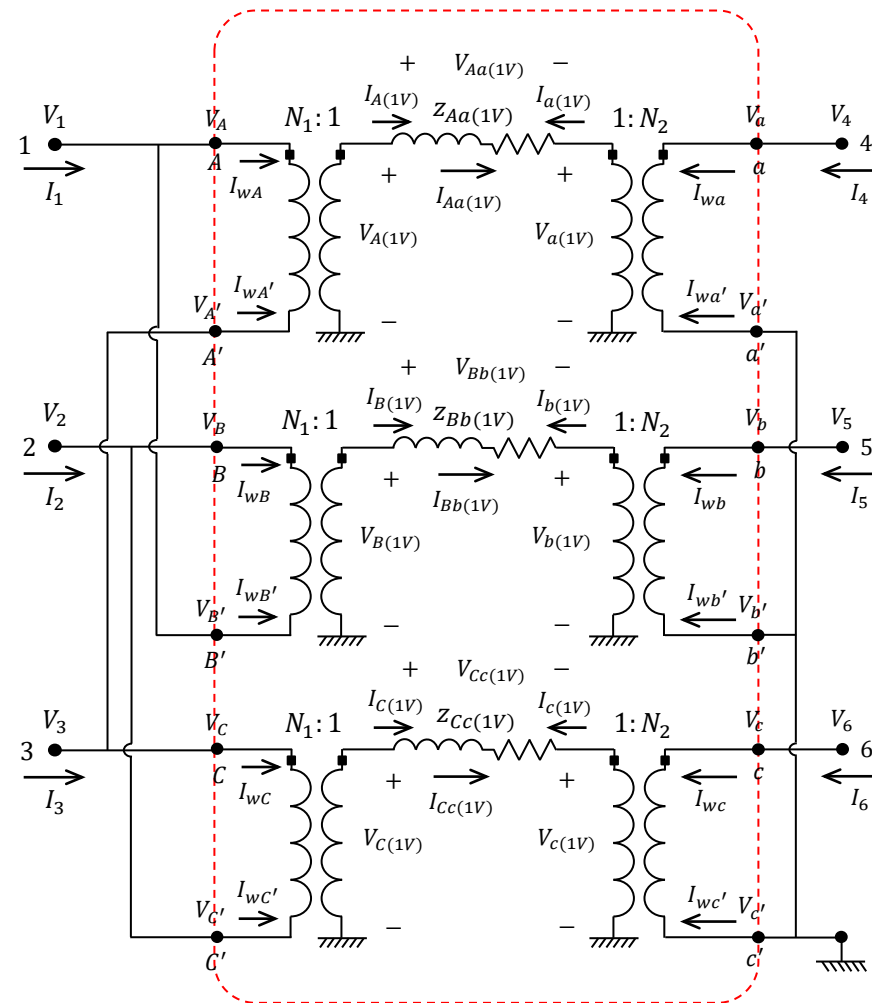
①

$$\begin{cases} I_{wA} = I_{A(1V)} / N_1 \\ I_{wA'} = -I_{A(1V)} / N_1 \\ I_{wa} = I_{a(1V)} / N_2 \\ I_{wa'} = -I_{a(1V)} / N_2 \\ I_{wB} = I_{B(1V)} / N_1 \\ I_{wB'} = -I_{B(1V)} / N_1 \\ I_{wb} = I_{b(1V)} / N_2 \\ I_{wb'} = -I_{b(1V)} / N_2 \\ I_{wC} = I_{C(1V)} / N_1 \\ I_{wC'} = -I_{C(1V)} / N_1 \\ I_{wc} = I_{c(1V)} / N_2 \\ I_{wc'} = -I_{c(1V)} / N_2 \end{cases} \Rightarrow \begin{bmatrix} I_{wA} \\ I_{wA'} \\ I_{wa} \\ I_{wa'} \\ I_{wB} \\ I_{wB'} \\ I_{wb} \\ I_{wb'} \\ I_{wC} \\ I_{wC'} \\ I_{wc} \\ I_{wc'} \end{bmatrix} = \begin{bmatrix} \frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{N_2} & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{N_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{N_1} & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{N_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{N_2} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{N_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{N_1} & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{N_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{N_2} \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{N_2} \end{bmatrix} \begin{bmatrix} I_{A(1V)} \\ I_{a(1V)} \\ I_{B(1V)} \\ I_{b(1V)} \\ I_{C(1V)} \\ I_{c(1V)} \end{bmatrix}$$

$$\Rightarrow I_w = N I_1, \text{ where } I_w = \begin{bmatrix} I_{wA} \\ I_{wA'} \\ I_{wa} \\ I_{wa'} \\ I_{wB} \\ I_{wB'} \\ I_{wb} \\ I_{wb'} \\ I_{wC} \\ I_{wC'} \\ I_{wc} \\ I_{wc'} \end{bmatrix}, N = \begin{bmatrix} \frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{N_2} & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{N_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{N_1} & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{N_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{N_2} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{N_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{N_1} & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{N_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{N_2} \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{N_2} \end{bmatrix}$$

Modeling A Transformer in OpenDSS

2. Three-phase Delta-Grounded Wye Step-down Transformer



2

$$\begin{cases} V_{wA} - V_{wA'} = V_{A(1V)}N_1 \\ V_{wa} - V_{wa'} = V_{a(1V)}N_2 \\ V_{wB} - V_{wB'} = V_{B(1V)}N_1 \\ V_{wb} - V_{wb'} = V_{b(1V)}N_2 \\ V_{wC} - V_{wC'} = V_{C(1V)}N_1 \\ V_{wc} - V_{wc'} = V_{c(1V)}N_2 \end{cases}$$



$$\begin{bmatrix} V_{A(1V)} \\ V_{a(1V)} \\ V_{B(1V)} \\ V_{b(1V)} \\ V_{C(1V)} \\ V_{c(1V)} \end{bmatrix} = \begin{bmatrix} \frac{1}{N_1} & -\frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{N_2} & -\frac{1}{N_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{N_1} & -\frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{N_2} & -\frac{1}{N_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{N_1} & -\frac{1}{N_1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{N_2} & -\frac{1}{N_2} \end{bmatrix} \begin{bmatrix} V_{wA} \\ V_{wA'} \\ V_{wa} \\ V_{wa'} \\ V_{wB} \\ V_{wB'} \\ V_{wb} \\ V_{wb'} \\ V_{wC} \\ V_{wC'} \\ V_{wc} \\ V_{wc'} \end{bmatrix}$$

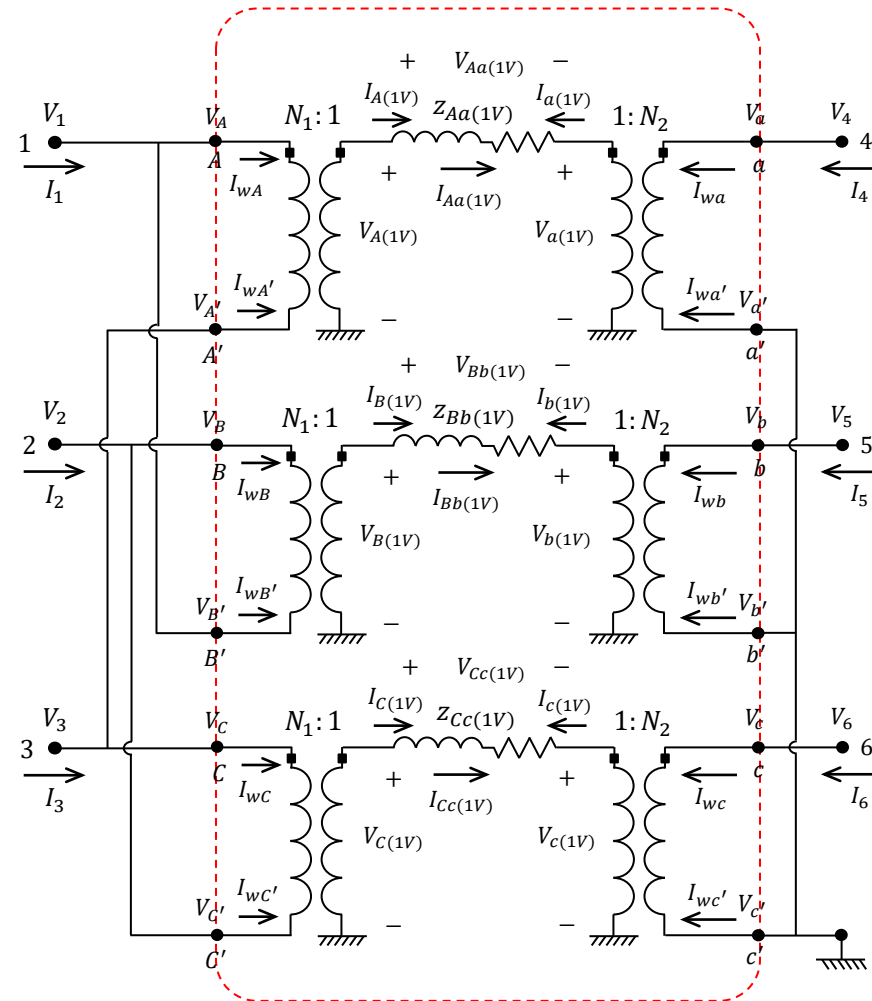


$$V_1 = N^T V_w, \text{ where } V_w = \begin{bmatrix} V_{wA} \\ V_{wA'} \\ V_{wa} \\ V_{wa'} \\ V_{wB} \\ V_{wB'} \\ V_{wb} \\ V_{wb'} \\ V_{wC} \\ V_{wC'} \\ V_{wc} \\ V_{wc'} \end{bmatrix}$$

Note: For a delta-connected winding, $N = VLL$; for a Y-connected winding, $N = \frac{VLL}{\sqrt{3}}$.

Modeling A Transformer in OpenDSS

2. Three-phase Delta-Grounded Wye Step-down Transformer



Next, let $I_w = Y_w V_w$. How to obtain Y_w ?

$$\left. \begin{aligned} I_w &= Y_w V_w \\ I_w &= N I_1 \\ V_1 &= N^T V_w \end{aligned} \right\}$$

$$\Rightarrow \left. \begin{aligned} N I_1 &= Y_w (N^T)^{-1} V_1 \\ I_1 &= Y_1 V_1 \end{aligned} \right\}$$

$$\Rightarrow N Y_1 V_1 = Y_w (N^T)^{-1} V_1$$

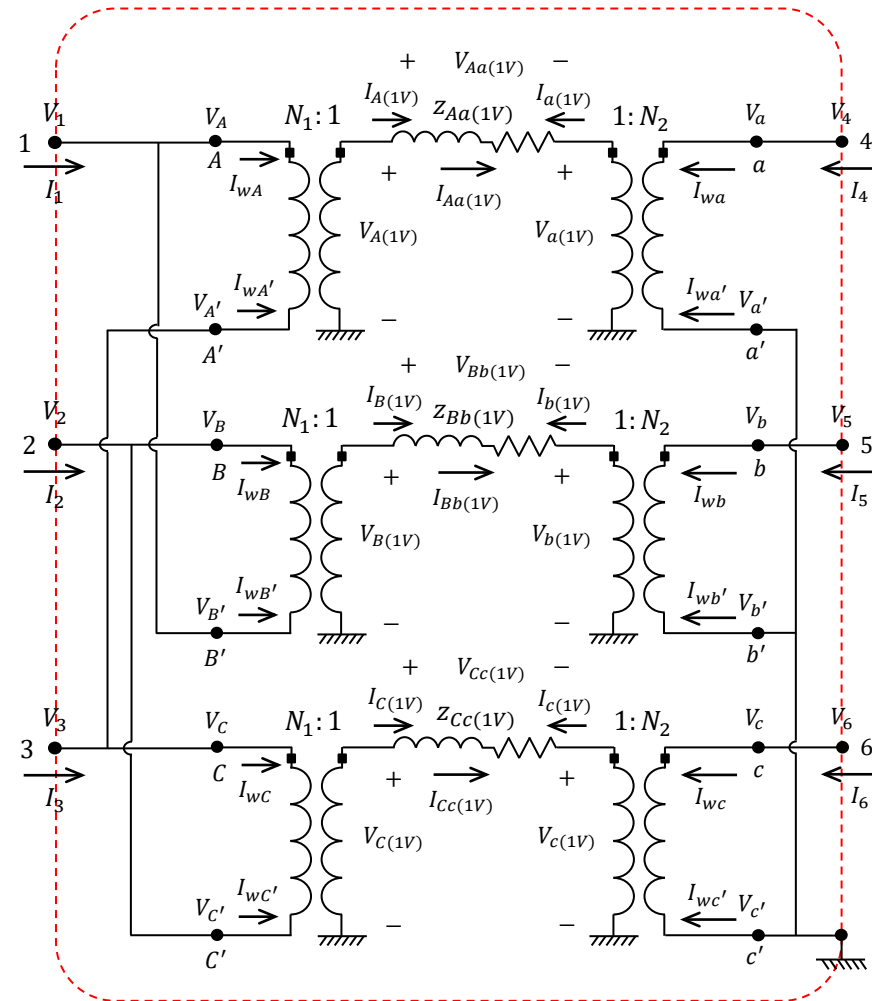
$$\Rightarrow \left. \begin{aligned} Y_w &= N Y_1 N^T \\ Y_1 &= B Y_{sc(1V)} B^T \end{aligned} \right\}$$

$$\Rightarrow Y_w = N B Y_{sc(1V)} B^T N^T$$

Modeling A Transformer in OpenDSS

2. Three-phase Delta-Grounded Wye Step-down Transformer

Step 3: Consider winding connections



$$\textcircled{1} \begin{cases} I_1 = I_{wA} + I_{wB'} \\ I_2 = I_{wB} + I_{wC'} \\ I_3 = I_{wC} + I_{wA'} \\ I_4 = I_{wa} \\ I_5 = I_{wb} \\ I_6 = I_{wc} \end{cases}$$

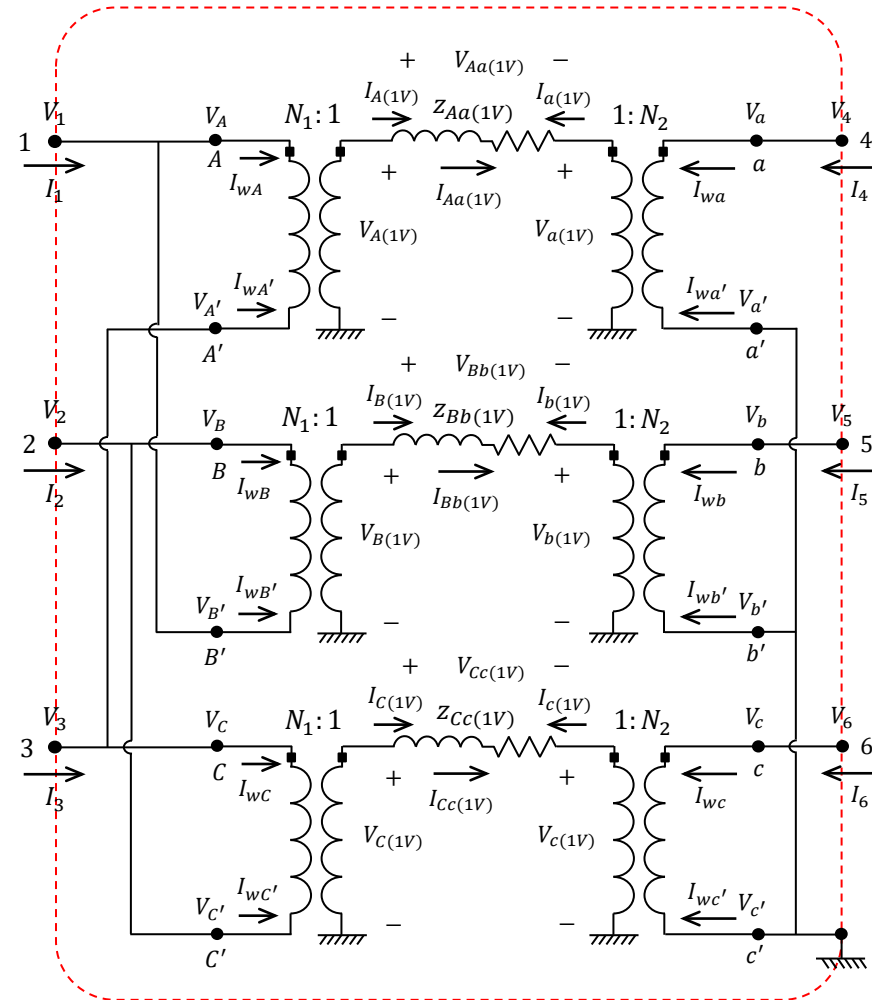


$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} I_{wA} \\ I_{wA'} \\ I_{wa} \\ I_{wA'} \\ I_{wB} \\ I_{wB'} \\ I_{wb} \\ I_{wb'} \\ I_{wC} \\ I_{wC'} \\ I_{wc} \\ I_{wc'} \end{bmatrix}$$

$$\Rightarrow I_{prim} = AI_w, \text{ where, } I_{prim} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix}, A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Modeling A Transformer in OpenDSS

2. Three-phase Delta-Grounded Wye Step-down Transformer

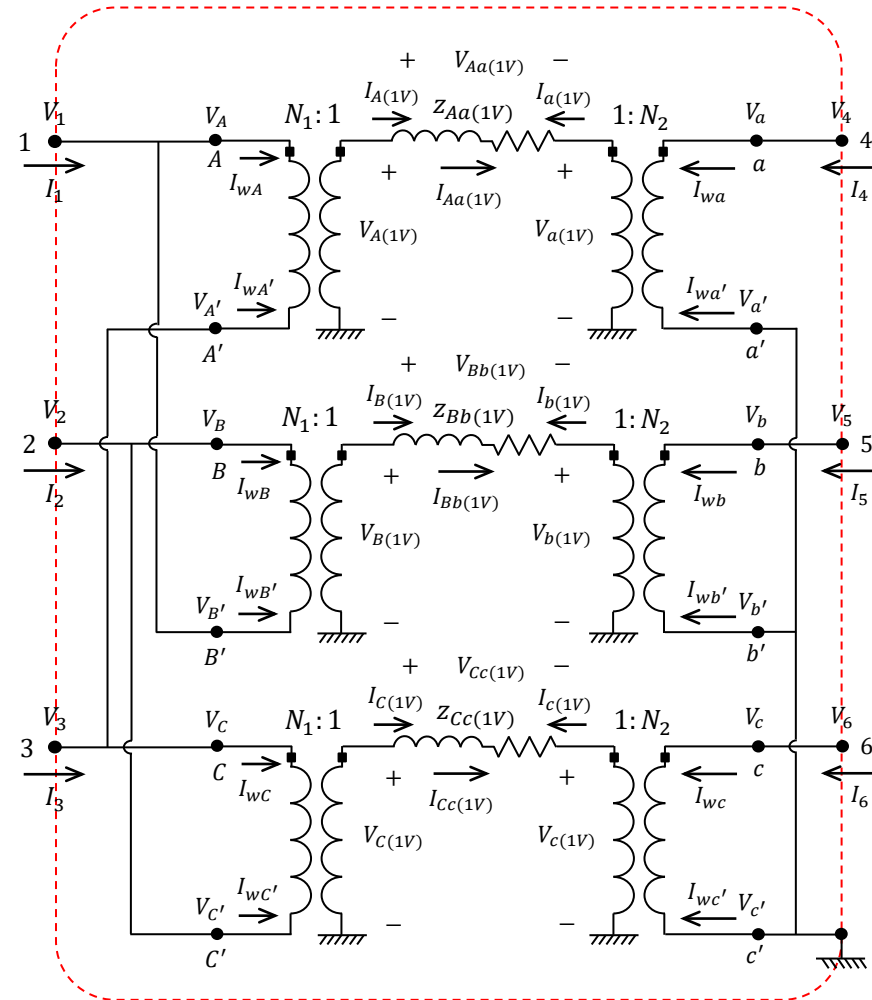


$$\begin{aligned}
 & \text{②} \left\{ \begin{aligned}
 V_A &= V_1 \\
 V_{A'} &= V_3 \\
 V_a &= V_4 \\
 V_{a'} &= 0 \\
 V_B &= V_2 \\
 V_{B'} &= V_1 \\
 V_b &= V_5 \\
 V_{b'} &= 0 \\
 V_C &= V_3 \\
 V_{C'} &= V_2 \\
 V_c &= V_6 \\
 V_{c'} &= 0
 \end{aligned} \right. \Rightarrow \begin{bmatrix} V_A \\ V_{A'} \\ V_a \\ V_{a'} \\ V_B \\ V_{B'} \\ V_b \\ V_{b'} \\ V_C \\ V_{C'} \\ V_c \\ V_{c'} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix}
 \end{aligned}$$

$$\Rightarrow \mathbf{V}_w = \mathbf{A}^T \mathbf{V}_{prim}, \text{ where } \mathbf{V}_{prim} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix}.$$

Modeling A Transformer in OpenDSS

2. Three-phase Delta-Grounded Wye Step-down Transformer



Let $I_{prim} = Y_{prim} V_{prim}$. How to obtain Y_{prim} ?

$$\left. \begin{aligned} I_{prim} &= Y_{prim} V_{prim} \\ I_{prim} &= A I_w \end{aligned} \right\}$$

$$\Rightarrow \left. \begin{aligned} A I_w &= Y_{prim} V_{prim} \\ V_w &= A^T V_{prim} \end{aligned} \right\}$$

$$\Rightarrow \left. \begin{aligned} A I_w &= Y_{prim} (A^T)^{-1} V_w \\ I_w &= Y_w V_w \end{aligned} \right\}$$

$$\Rightarrow \left. \begin{aligned} A Y_w V_w &= Y_{prim} (A^T)^{-1} V_w \\ Y_w &= N B Y_{sc(1V)} B^T N^T \end{aligned} \right\}$$

$$\Rightarrow Y_{prim} = A N B Y_{sc(1V)} B^T N^T A^T$$

Modeling A Transformer in OpenDSS

2. Three-phase Delta-Grounded Wye Step-down Transformer

An example:

// Define a three-phase transformer.

New Transformer.xfmr phases=3 windings=2 xhl=5

~ wdg=1 bus=K.1.2.3 conn=delta kV=115 kva=500 %r=1

~ wdg=2 bus=M.1.2.3.0 conn=weye kV=4.16 kva=500 %r=1

Specify which winding
will be edited

Winding
connection

Rated voltage

Base kVA
rating

Percent
resistance

Computing Y_{prim} using Matlab:

$$Y_{Aa(1V)} = Y_{Bb(1V)} = Y_{Cc(1V)} = \frac{1}{[r(pu) + jx(pu)] * \frac{1^2}{S_t/3}} = \frac{500000}{3 * (0.02 + j0.05)} = 1149425.2873 - j2873563.2184 S$$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix}$$

$$N^T = \begin{bmatrix} \frac{1}{115000} & -\frac{1}{115000} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4160/\sqrt{3}} & -\frac{1}{4160/\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{115000} & -\frac{1}{115000} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4160/\sqrt{3}} & -\frac{1}{4160/\sqrt{3}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{115000} & -\frac{1}{115000} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4160/\sqrt{3}} & -\frac{1}{4160/\sqrt{3}} \end{bmatrix}$$

19

Modeling A Transformer in OpenDSS

2. Three-phase Delta-Grounded Wye Step-down Transformer

$$\mathbf{Y}_{sc(1V)} = \begin{bmatrix} y_{Aa(1V)} & 0 & 0 \\ 0 & y_{Bb(1V)} & 0 \\ 0 & 0 & y_{Cc(1V)} \end{bmatrix} = \begin{bmatrix} 1149425.2873 - j2873563.2184 & 0 & 0 \\ 0 & 1149425.2873 - j2873563.2184 & 0 \\ 0 & 0 & 1149425.2873 - j2873563.2184 \end{bmatrix}$$

Calculate $\mathbf{Y}_{prim} = \mathbf{A}\mathbf{N}\mathbf{B}\mathbf{Y}_{sc(1V)}\mathbf{B}^T\mathbf{N}^T\mathbf{A}^T =$

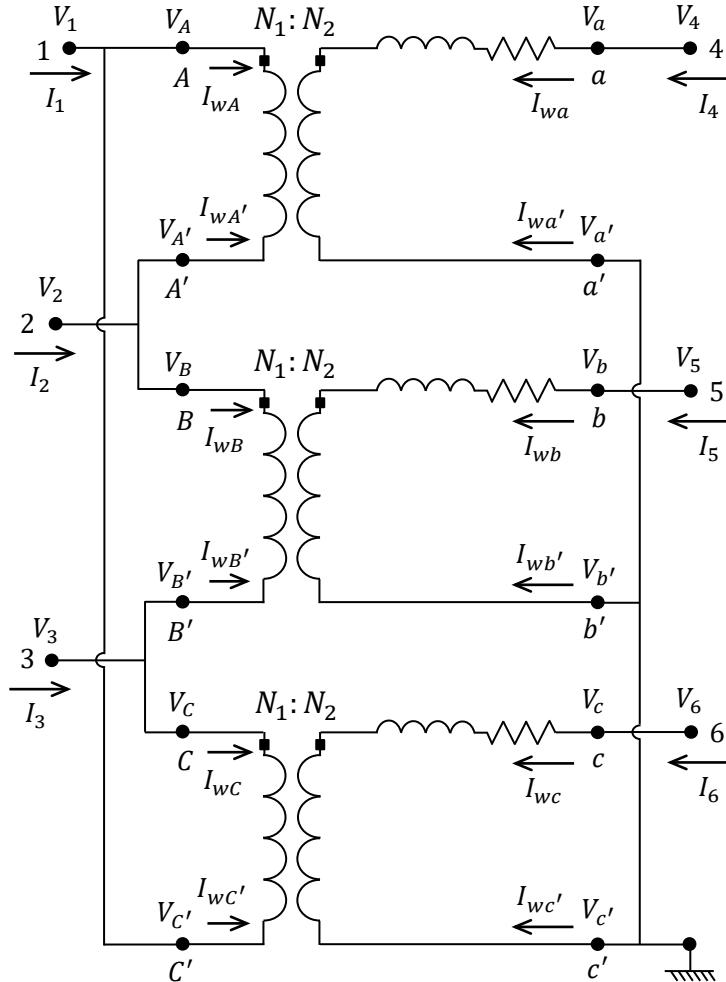
$$\begin{bmatrix} 0.00017383 - j0.00043457 & -8.6913e-05 + j0.00021728 & -8.6913e-05 + j0.00021728 & -0.00416150 + j0.01040376 & 0.00416150 - j0.01040376 & 0 + j0 \\ -8.6913e-05 + j0.00021728 & 0.00017383 - j0.00043457 & -8.6913e-05 + j0.00021728 & 0 + j0 & -0.00416150 + j0.01040376 & 0.00416150 - j0.01040376 \\ -8.6913e-05 + j0.00021728 & -8.6913e-05 + j0.00021728 & 0.00017383 - j0.00043457 & 0.00416150 - j0.01040376 & 0 + j0 & -0.00416150 + j0.01040376 \\ -0.00416150 + j0.01040376 & 0 + j0 & 0.000260739 - j0.00065184 & 0.19925780 - j0.49814451 & 0 + j0 & 0 + j0 \\ 0.00416150 - j0.01040376 & -0.00416150 + j0.01040376 & 0 + j0 & 0 + j0 & 0.19925780 - j0.49814451 & 0 + j0 \\ 0 + j0 & 0.00416150 - j0.01040376 & -0.00416150 + j0.01040376 & 0 + j0 & 0.19925780 - j0.49814451 & 0.19925780 - j0.49814451 \end{bmatrix}$$

Exported $\mathbf{Y}_{prim}$ from OpenDSS:

	G						B					
	↓						↓					
1	0.000174	-0.00043	-8.69E-05	0.000217	-8.69E-05	0.000217	-0.00416	0.010404	0.004162	-0.0104	0	0
2	-8.69E-05	0.000217	0.000174	-0.00043	-8.69E-05	0.000217	0	0	-0.00416	0.010404	0.004162	-0.0104
3	-8.69E-05	0.000217	-8.69E-05	0.000217	0.000174	-0.00043	0.004162	-0.0104	0	0	-0.00416	0.010404
4	-0.00416	0.010404	0	0	0.004162	-0.0104	0.199258	-0.49814	0	0	0	0
5	0.004162	-0.0104	-0.00416	0.010404	0	0	0	0	0.199258	-0.49814	0	0
6	0	0	0.004162	-0.0104	-0.00416	0.010404	0	0	0	0	0.199258	-0.49814
	1	2	3	4	5	6						

Modeling A Transformer in OpenDSS

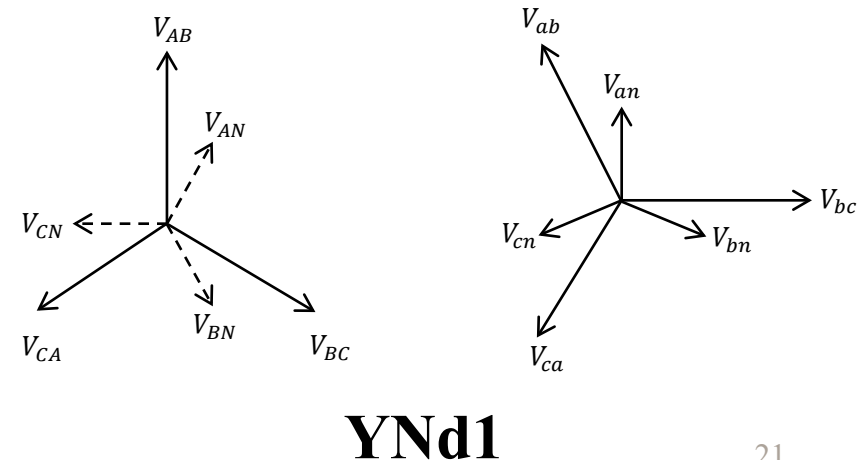
3. Three-phase Delta-Grounded Wye Step-up Transformer



As the IEC60076-1 standard has stated, the notation is **HV-LV** in sequence. Therefore, this step-up transformer with a delta-connected primary, and star-connected secondary, is not written as 'dY1', but 'Yd1'. The 1 indicates the LV winding lags the HV by 30 degrees.

Note that

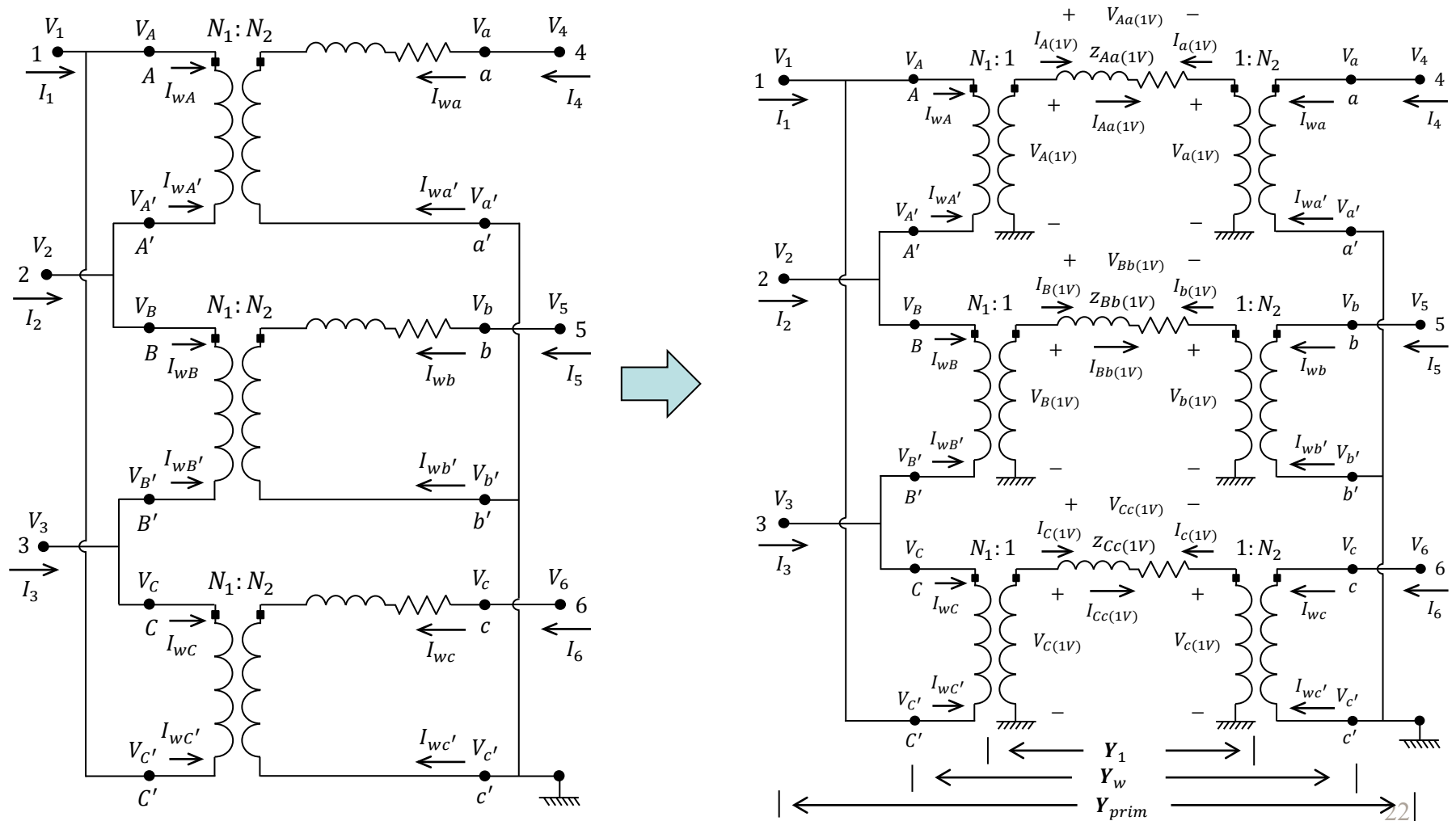
- The HV side always comes before the LV side, regardless of which is the primary winding.
- HV winding is taken as a reference.
- Phase rotation is always counterclockwise.
- 1 = 30°, 2 = 60°, 3 = 90°, 6 = 180° and 12 = 0° or 360°.



https://en.wikipedia.org/wiki/Vector_group

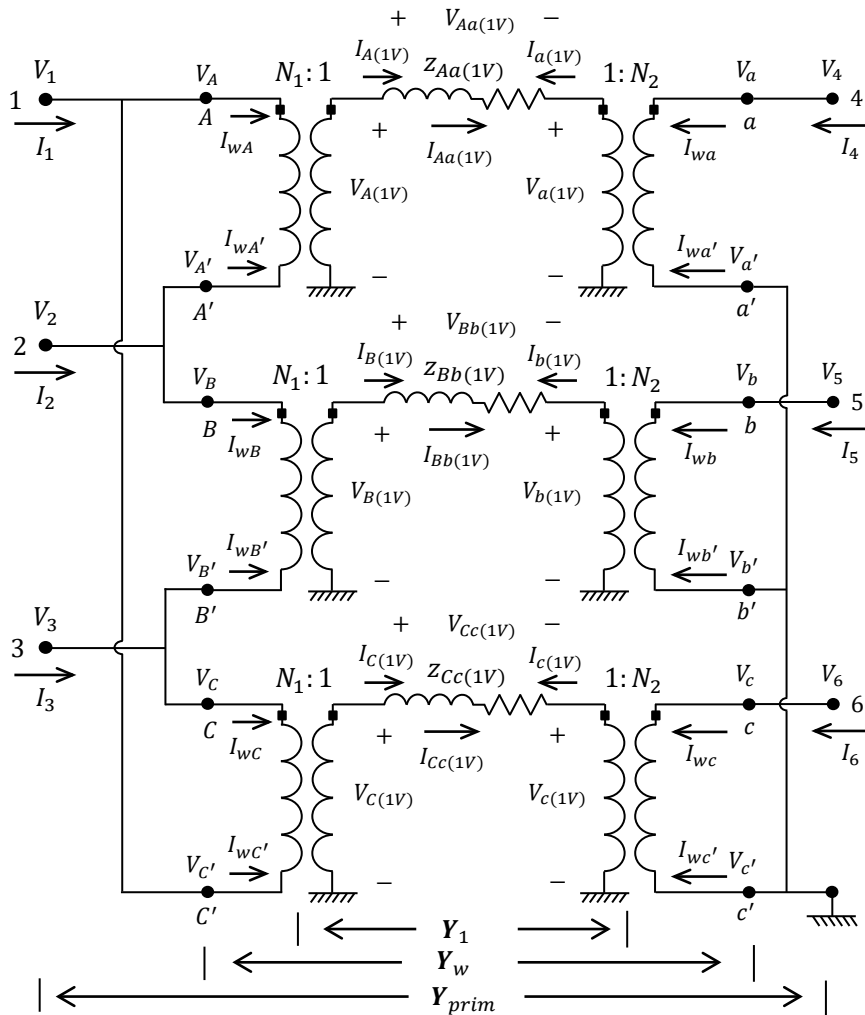
Modeling A Transformer in OpenDSS

3. Three-phase Delta-Grounded Wye Step-up Transformer



Modeling A Transformer in OpenDSS

3. Three-phase Delta-Grounded Wye Step-up Transformer



Note:

Compared to the 3-phase delta-grounded wye step-down transformer, the equation, $Y_{prim} = ANBy_{sc(1V)} B^T N^T A^T$, still applies to a 3-phase delta-grounded wye step-up transformer. The only difference exists in the formation of matrix A due to a different connection of delta winding, and the values of N_1 and N_2 . Specifically, according to the figure on the left, we have

$$\begin{cases} I_1 = I_{wA} + I_{wC'} \\ I_2 = I_{wB} + I_{wA'} \\ I_3 = I_{wC} + I_{wB'} \\ I_4 = I_{wa} \\ I_5 = I_{wb} \\ I_6 = I_{wc} \end{cases} \Rightarrow \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} I_{wA} \\ I_{wA'} \\ I_{wa} \\ I_{wA'} \\ I_{wB} \\ I_{wB'} \\ I_{wb} \\ I_{wb'} \\ I_{wC} \\ I_{wC'} \\ I_{wc} \\ I_{wc'} \end{bmatrix}$$

$$\Rightarrow A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Modeling A Transformer in OpenDSS

3. Three-phase Delta-Grounded Wye Step-up Transformer

An example:

// Define a three-phase transformer.

New Transformer.xfmr phases=3 windings=2 xhl=5

~ wdg=1 bus=K.1.2.3 conn=delta kV=4.16 kva=500 %r=1

~ wdg=2 bus=M.1.2.3.0 conn=wye kV=115 kva=500 %r=1

Specify which winding
will be edited

Winding
connection

Rated voltage

Base kVA
rating

Percent
resistance

Computing Y_{prim} using Matlab:

$$Y_{Aa(1V)} = Y_{Bb(1V)} = Y_{Cc(1V)} = \frac{1}{[r(pu) + jx(pu)] * \frac{1^2}{S_t/3}} = \frac{500000}{3 * (0.02 + j0.05)} = 1149425.2873 - j2873563.2184$$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix}$$

$$N^T = \begin{bmatrix} \frac{1}{4160} & -\frac{1}{4160} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{115000/\sqrt{3}} & -\frac{1}{115000/\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{4160} & -\frac{1}{4160} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{115000/\sqrt{3}} & -\frac{1}{115000/\sqrt{3}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4160} & -\frac{1}{4160} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{115000/\sqrt{3}} & -\frac{1}{115000/\sqrt{3}} \end{bmatrix}$$

24

Modeling A Transformer in OpenDSS

3. Three-phase Delta-Grounded Wye Step-up Transformer

$$\mathbf{Y}_{sc(1V)} = \begin{bmatrix} y_{Aa(1V)} & 0 & 0 \\ 0 & y_{Bb(1V)} & 0 \\ 0 & 0 & y_{Cc(1V)} \end{bmatrix} = \begin{bmatrix} 1149425.2873 - j2873563.2184 & 0 & 0 \\ 0 & 1149425.2873 - j2873563.2184 & 0 \\ 0 & 0 & 1149425.2873 - j2873563.2184 \end{bmatrix}$$

Calculate $\mathbf{Y}_{prim} = \mathbf{A}\mathbf{N}\mathbf{B}\mathbf{Y}_{sc(1V)}\mathbf{B}^T\mathbf{N}^T\mathbf{A}^T =$

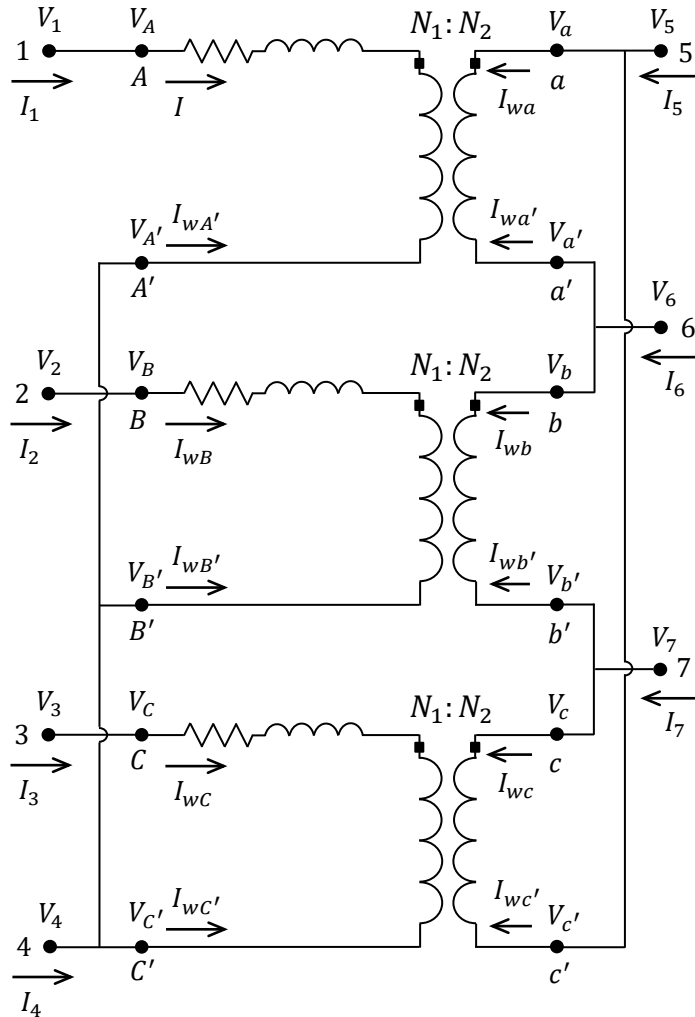
$$\begin{bmatrix} 0.13283854 - j0.33209634 & -0.06641927 + j0.16604817 & -0.06641927 + j0.16604817 & -0.00416150 + j0.01040376 & 0 + j0 & 0.00416150 - j0.01040376 \\ -0.06641927 + j0.16604817 & 0.13283854 - j0.33209634 & -0.06641927 + j0.16604817 & 0.00416150 - j0.01040376 & -0.00416150 + j0.01040376 & 0 + j0 \\ -0.06641927 + j0.16604817 & -0.06641927 + j0.16604817 & 0.13283854 - j0.33209634 & 0 + j0 & 0.00416150 - j0.01040376 & -0.00416150 + j0.01040376 \\ -0.00416150 + j0.01040376 & 0.00416150 - j0.01040376 & 0 + j0 & 0.00026074 - j0.00065185 & 0 + j0 & 0 + j0 \\ 0 + j0 & -0.00416150 + j0.01040376 & 0.00416150 - j0.01040376 & 0 + j0 & 0.00026074 - j0.00065185 & 0 + j0 \\ 0.00416150 - j0.01040376 & 0 + j0 & -0.00416150 + j0.01040376 & 0 + j0 & 0 + j0 & 0.00026074 - j0.00065185 \end{bmatrix}$$

Exported $\mathbf{Y}_{prim}$ from OpenDSS:

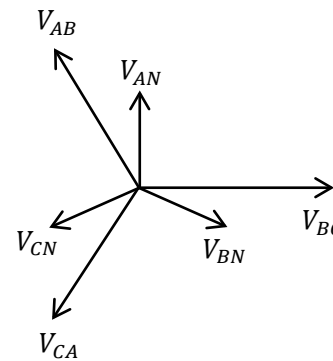
	G		B									
	↓		↓									
1	0.132839	-0.3321	-0.06642	0.166048	-0.06642	0.166048	-0.00416	0.010404	0	0	0.004162	-0.0104
2	-0.06642	0.166048	0.132839	-0.3321	-0.06642	0.166048	0.004162	-0.0104	-0.00416	0.010404	0	0
3	-0.06642	0.166048	-0.06642	0.166048	0.132839	-0.3321	0	0	0.004162	-0.0104	-0.00416	0.010404
4	-0.00416	0.010404	0.004162	-0.0104	0	0	0.000261	-0.00065	0	0	0	0
5	0	0	-0.00416	0.010404	0.004162	-0.0104	0	0	0.000261	-0.00065	0	0
6	0.004162	-0.0104	0	0	-0.00416	0.010404	0	0	0	0	0.000261	-0.00065
	1		2		3		4		5		6	

Modeling A Transformer in OpenDSS

4. Three-phase Ungrounded Wye-Delta Step-down Transformer



- The HV side always comes before the LV side, regardless of which is the primary winding.
- HV winding is taken as a reference.
- Phase rotation is always counterclockwise.
- 1 = 30°, 2 = 60°, 3 = 90°, 6 = 180° and 12 = 0° or 360°.

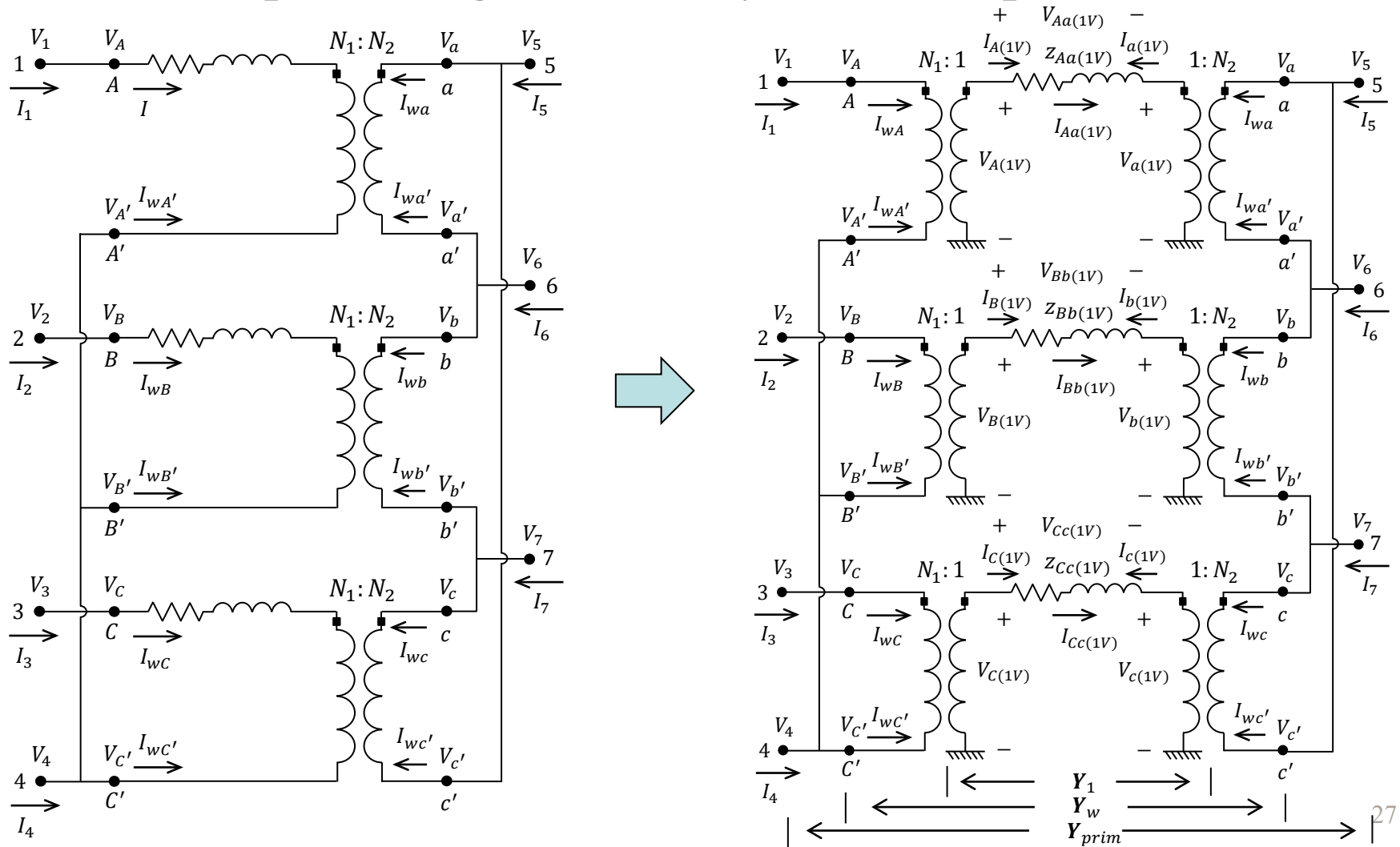


YNd1

26

Modeling A Transformer in OpenDSS

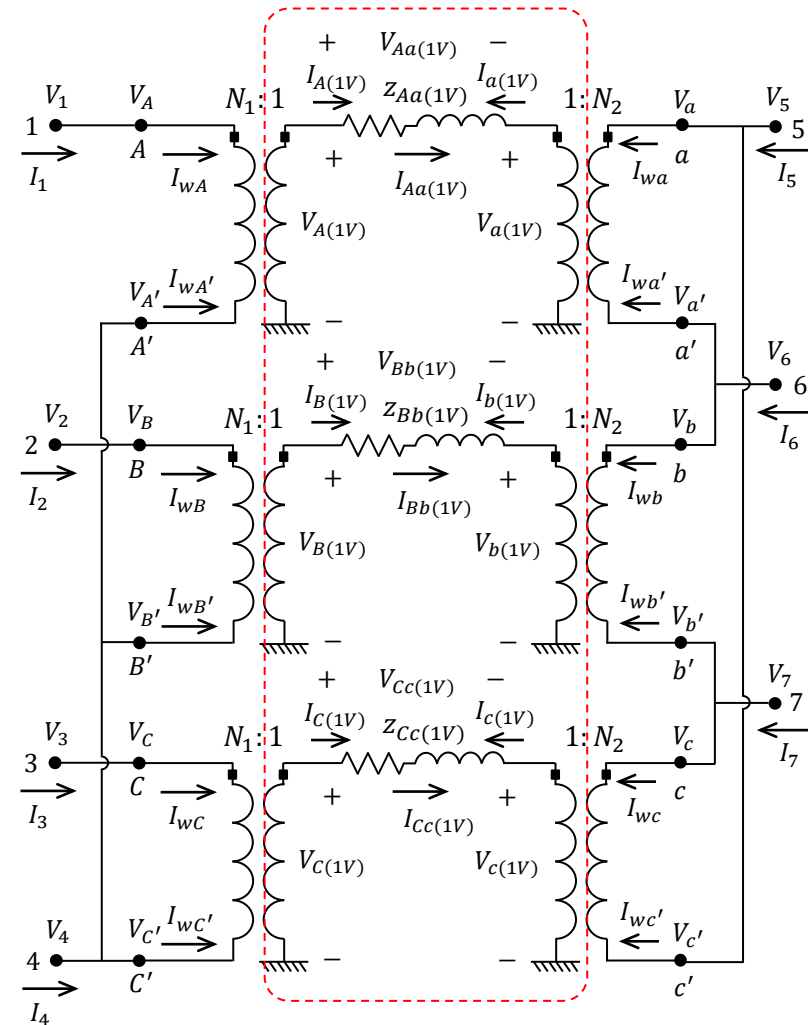
4. Three-phase Ungrounded Wye-Delta Step-down Transformer



Modeling A Transformer in OpenDSS

4. Three-phase Ungrounded Wye-Delta Step-down Transformer

Step 1: Compute the nodal admittance matrix on a one turn or one voltage base.



$$\textcircled{1} \quad \begin{cases} I_A(1V) = I_{Aa}(1V) \\ I_a(1V) = -I_{Aa}(1V) \\ I_B(1V) = I_{Bb}(1V) \\ I_b(1V) = -I_{Bb}(1V) \\ I_C(1V) = I_{Cc}(1V) \\ I_c(1V) = -I_{Cc}(1V) \end{cases} \Rightarrow \begin{bmatrix} I_A(1V) \\ I_a(1V) \\ I_B(1V) \\ I_b(1V) \\ I_C(1V) \\ I_c(1V) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} I_{Aa}(1V) \\ I_{Bb}(1V) \\ I_{Cc}(1V) \end{bmatrix}$$

$$\Rightarrow \mathbf{I}_1 = \mathbf{B} \mathbf{I}_{sc(1V)}, \text{ where } \mathbf{I}_1 = \begin{bmatrix} I_A(1V) \\ I_a(1V) \\ I_B(1V) \\ I_b(1V) \\ I_C(1V) \\ I_c(1V) \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix}, \mathbf{I}_{sc(1V)} = \begin{bmatrix} I_{Aa}(1V) \\ I_{Bb}(1V) \\ I_{Cc}(1V) \end{bmatrix}$$

$$\textcircled{2} \quad \begin{cases} V_{Aa}(1V) = V_A(1V) - V_a(1V) \\ V_{Bb}(1V) = V_B(1V) - V_b(1V) \\ V_{Cc}(1V) = V_C(1V) - V_c(1V) \end{cases} \Rightarrow \begin{bmatrix} V_{Aa}(1V) \\ V_{Bb}(1V) \\ V_{Cc}(1V) \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} V_A(1V) \\ V_a(1V) \\ V_B(1V) \\ V_b(1V) \\ V_C(1V) \\ V_c(1V) \end{bmatrix}$$

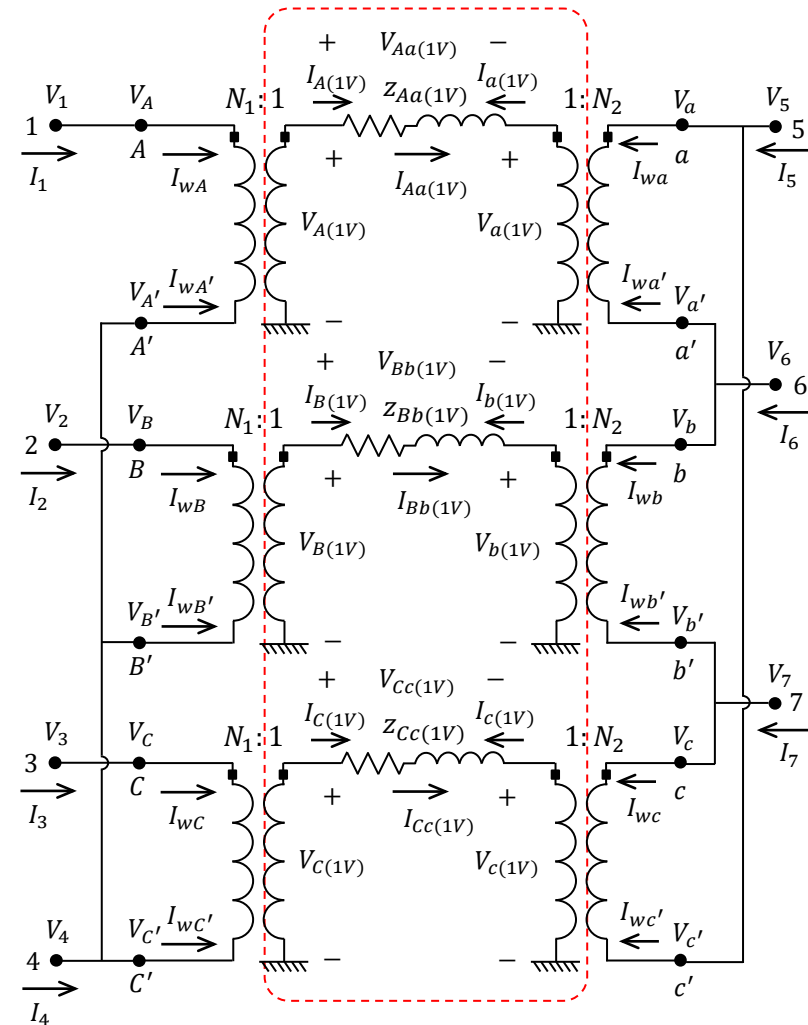
$$\Rightarrow \mathbf{V}_{sc(1V)} = \mathbf{B}^T \mathbf{V}_1, \text{ where } \mathbf{V}_{sc(1V)} = \begin{bmatrix} V_{Aa}(1V) \\ V_{Bb}(1V) \\ V_{Cc}(1V) \end{bmatrix}, \mathbf{V}_1 = \begin{bmatrix} V_A(1V) \\ V_a(1V) \\ V_B(1V) \\ V_b(1V) \\ V_C(1V) \\ V_c(1V) \end{bmatrix}$$

$$\textcircled{3} \quad \begin{cases} V_{Aa}(1V) = Z_{Aa}(1V) I_{Aa}(1V) \\ V_{Bb}(1V) = Z_{Bb}(1V) I_{Bb}(1V) \\ V_{Cc}(1V) = Z_{Cc}(1V) I_{Cc}(1V) \end{cases} \Rightarrow \begin{bmatrix} V_{Aa}(1V) \\ V_{Bb}(1V) \\ V_{Cc}(1V) \end{bmatrix} = \begin{bmatrix} Z_{Aa}(1V) & 0 & 0 \\ 0 & Z_{Bb}(1V) & 0 \\ 0 & 0 & Z_{Cc}(1V) \end{bmatrix} \begin{bmatrix} I_{Aa}(1V) \\ I_{Bb}(1V) \\ I_{Cc}(1V) \end{bmatrix}$$

$$\Rightarrow \mathbf{V}_{sc(1V)} = \mathbf{Z}_{sc(1V)} \mathbf{I}_{sc(1V)} \Rightarrow \mathbf{Y}_{sc(1V)} \mathbf{V}_{sc(1V)} = \mathbf{I}_{sc(1V)}, \text{ where } \mathbf{Y}_{sc(1V)} = (\mathbf{Z}_{sc(1V)})^{-1}$$

Modeling A Transformer in OpenDSS

4. Three-phase Ungrounded Wye-Delta Step-down Transformer



Let $I_1 = Y_1 V_1$, where Y_1 is the admittance matrix on a one turn or one voltage base. What is Y_1 ?

$$\left. \begin{aligned} I_1 &= Y_1 V_1 \\ I_1 &= B I_{sc(1V)} \end{aligned} \right\}$$

$$\Rightarrow \left. \begin{aligned} B I_{sc(1V)} &= Y_1 V_1 \\ V_{sc(1V)} &= B^T V_1 \end{aligned} \right\}$$

$$\Rightarrow \left. \begin{aligned} B I_{sc(1V)} &= Y_1 (B^T)^{-1} V_{sc(1V)} \\ I_{sc(1V)} &= Y_{sc(1V)} V_{sc(1V)} \end{aligned} \right\}$$

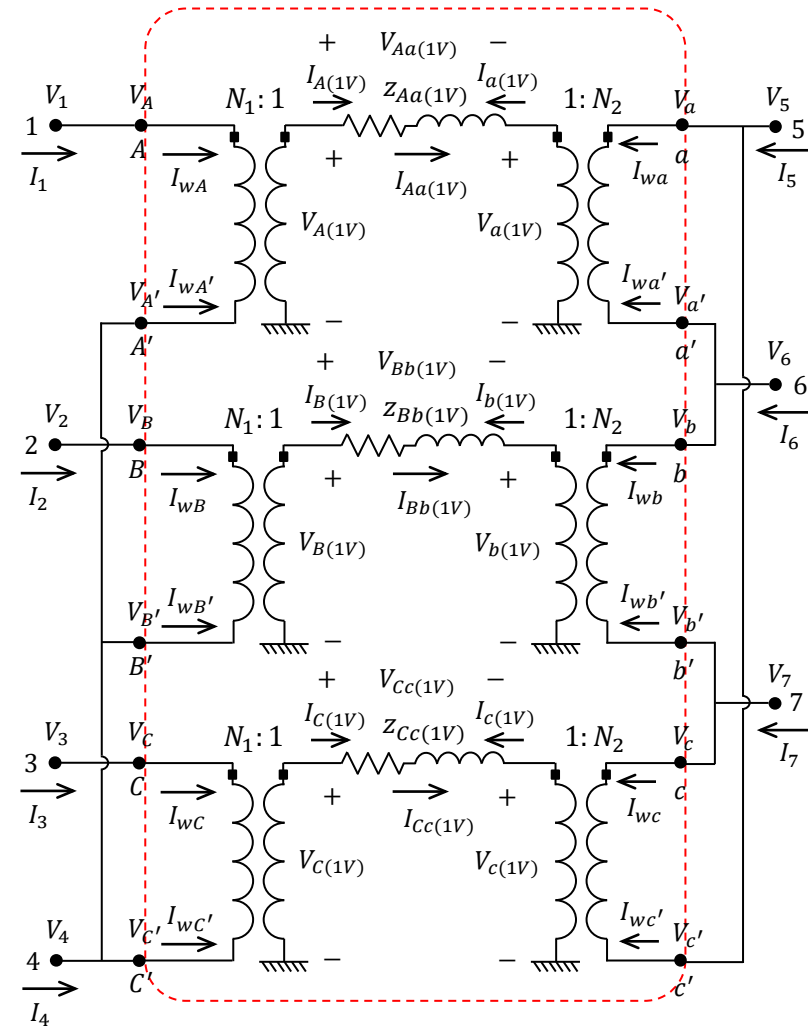
$$\Rightarrow B Y_{sc(1V)} V_{sc(1V)} = Y_1 (B^T)^{-1} V_{sc(1V)}$$

$$\Rightarrow Y_1 = B Y_{sc(1V)} B^T$$

Modeling A Transformer in OpenDSS

4. Three-phase Ungrounded Wye-Delta Step-down Transformer

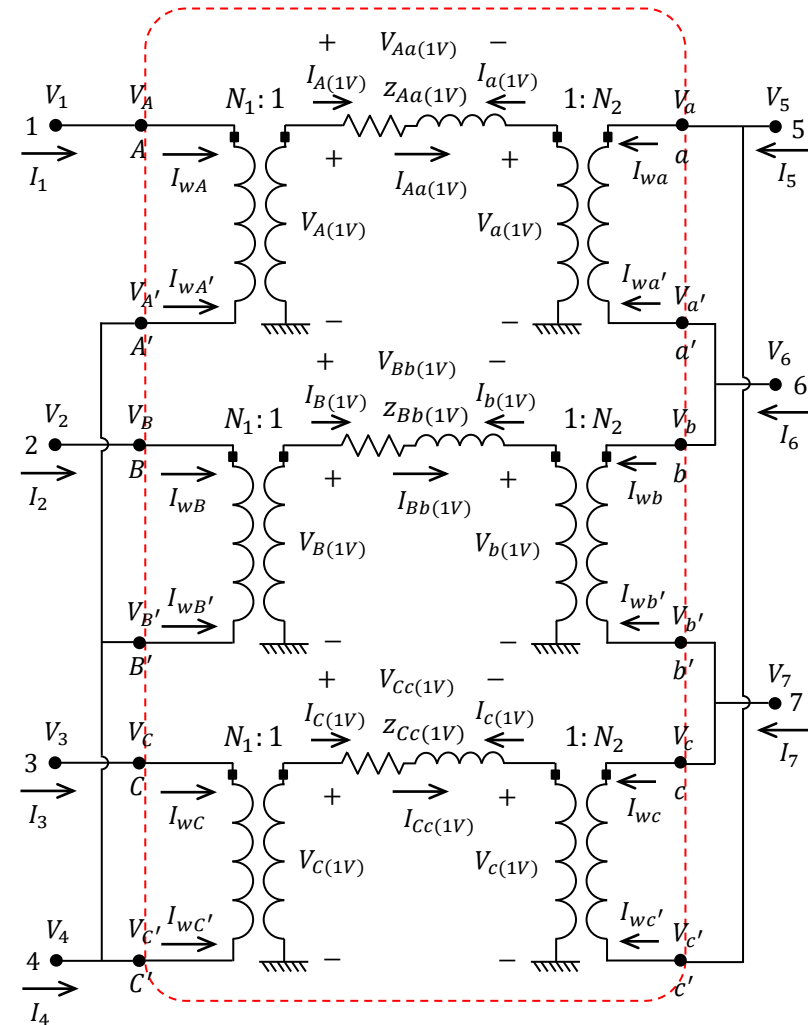
Step 2: Consider winding turns ratio.



$$\begin{aligned}
 & \text{①} \left\{ \begin{aligned} I_{wA} &= I_{A(1V)} / N_1 \\ I_{wA'} &= -I_{A(1V)} / N_1 \\ I_{wa} &= I_{a(1V)} / N_2 \\ I_{wa'} &= -I_{a(1V)} / N_2 \\ I_{wB} &= I_{B(1V)} / N_1 \\ I_{wB'} &= -I_{B(1V)} / N_1 \\ I_{wb} &= I_{b(1V)} / N_2 \\ I_{wb'} &= -I_{b(1V)} / N_2 \\ I_{wC} &= I_{C(1V)} / N_1 \\ I_{wC'} &= -I_{C(1V)} / N_1 \\ I_{wc} &= I_{c(1V)} / N_2 \\ I_{wc'} &= -I_{c(1V)} / N_2 \end{aligned} \right. \Rightarrow \begin{bmatrix} I_{wA} \\ I_{wA'} \\ I_{wa} \\ I_{wa'} \\ I_{wB} \\ I_{wB'} \\ I_{wb} \\ I_{wb'} \\ I_{wC} \\ I_{wC'} \\ I_{wc} \\ I_{wc'} \end{bmatrix} = \begin{bmatrix} \frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{N_2} & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{N_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{N_1} & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{N_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{N_2} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{N_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{N_1} & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{N_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{N_2} \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{N_2} \end{bmatrix} \begin{bmatrix} I_{A(1V)} \\ I_{a(1V)} \\ I_{B(1V)} \\ I_{b(1V)} \\ I_{C(1V)} \\ I_{c(1V)} \end{bmatrix} \\
 & \Rightarrow I_w = N I_1, \text{ where } I_w = \begin{bmatrix} I_{wA} \\ I_{wA'} \\ I_{wa} \\ I_{wa'} \\ I_{wB} \\ I_{wB'} \\ I_{wb} \\ I_{wb'} \\ I_{wC} \\ I_{wC'} \\ I_{wc} \\ I_{wc'} \end{bmatrix}, N = \begin{bmatrix} \frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{N_2} & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{N_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{N_1} & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{N_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{N_2} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{N_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{N_1} & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{N_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{N_2} \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{N_2} \end{bmatrix}
 \end{aligned}$$

Modeling A Transformer in OpenDSS

4. Three-phase Ungrounded Wye-Delta Step-down Transformer



②

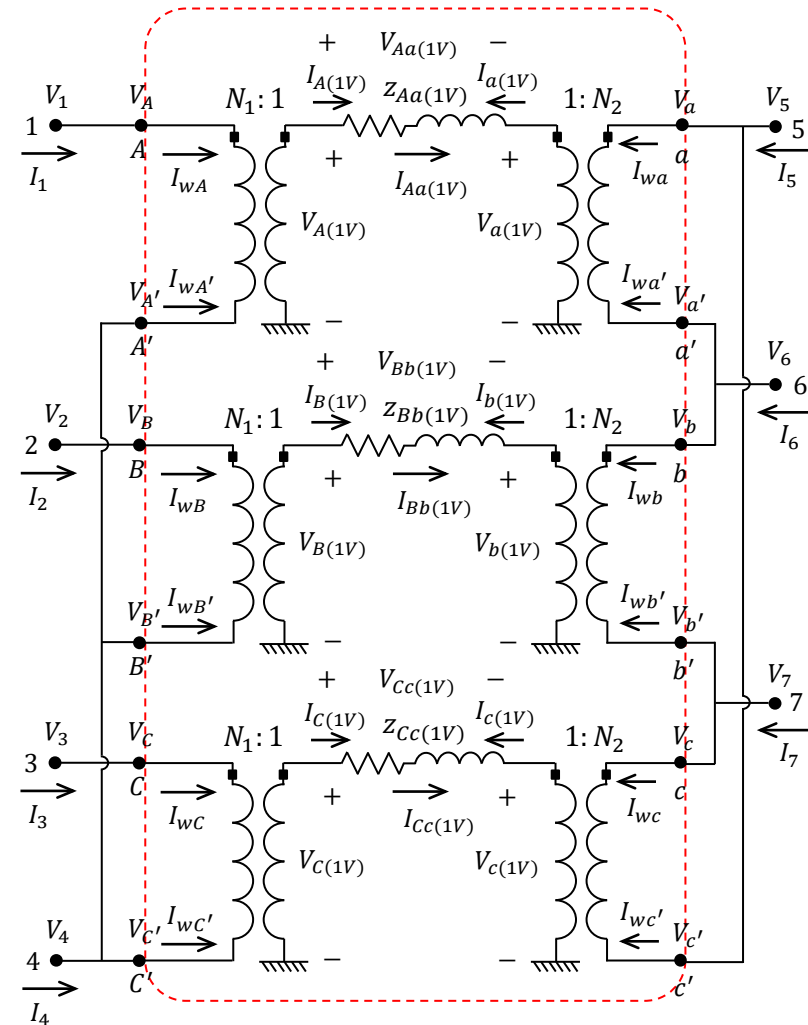
$$\begin{cases} V_{wA} - V_{wA'} = V_{A(1V)}N_1 \\ V_{wa} - V_{wa'} = V_{a(1V)}N_2 \\ V_{wB} - V_{wB'} = V_{B(1V)}N_1 \\ V_{wb} - V_{wb'} = V_{b(1V)}N_2 \\ V_{wC} - V_{wC'} = V_{C(1V)}N_1 \\ V_{wc} - V_{wc'} = V_{c(1V)}N_2 \end{cases}$$

$$\begin{bmatrix} V_{A(1V)} \\ V_{a(1V)} \\ V_{B(1V)} \\ V_{b(1V)} \\ V_{C(1V)} \\ V_{c(1V)} \end{bmatrix} = \begin{bmatrix} \frac{1}{N_1} & -\frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{N_2} & -\frac{1}{N_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{N_1} & -\frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{N_2} & -\frac{1}{N_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{N_1} & -\frac{1}{N_1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{N_2} & -\frac{1}{N_2} \end{bmatrix} \begin{bmatrix} V_{wA} \\ V_{wA'} \\ V_{wa} \\ V_{wa'} \\ V_{wB} \\ V_{wB'} \\ V_{wb} \\ V_{wb'} \\ V_{wC} \\ V_{wC'} \\ V_{wc} \\ V_{wc'} \end{bmatrix}$$

$$\Rightarrow V_1 = N^T V_w, \text{ where } V_w = \begin{bmatrix} V_{wA} \\ V_{wA'} \\ V_{wa} \\ V_{wa'} \\ V_{wB} \\ V_{wB'} \\ V_{wb} \\ V_{wb'} \\ V_{wC} \\ V_{wC'} \\ V_{wc} \\ V_{wc'} \end{bmatrix}$$

Modeling A Transformer in OpenDSS

4. Three-phase Ungrounded Wye-Delta Step-down Transformer



Next, let $I_w = Y_w V_w$. How to obtain Y_w ?

$$\left. \begin{aligned} I_w &= Y_w V_w \\ I_w &= N I_1 \\ V_1 &= N^T V_w \end{aligned} \right\}$$

$$\Rightarrow \left. \begin{aligned} N I_1 &= Y_w (N^T)^{-1} V_1 \\ I_1 &= Y_1 V_1 \end{aligned} \right\}$$

$$\Rightarrow N Y_1 V_1 = Y_w (N^T)^{-1} V_1$$

$$\Rightarrow \left. \begin{aligned} Y_w &= N Y_1 N^T \\ Y_1 &= B Y_{sc(1V)} B^T \end{aligned} \right\}$$

$$\Rightarrow Y_w = N B Y_{sc(1V)} B^T N^T$$

Modeling A Transformer in OpenDSS

4. Three-phase Ungrounded Wye-Delta Step-down Transformer

Step 3: Consider winding connections

①

$$\begin{cases} I_1 = I_{wA} \\ I_2 = I_{wB} \\ I_3 = I_{wC} \\ I_4 = I_{wA'} + I_{wB'} + I_{wC'} \\ I_5 = I_{wa} + I_{wc'} \\ I_6 = I_{wa'} + I_{wb} \\ I_7 = I_{wb'} + I_{wc} \end{cases}$$



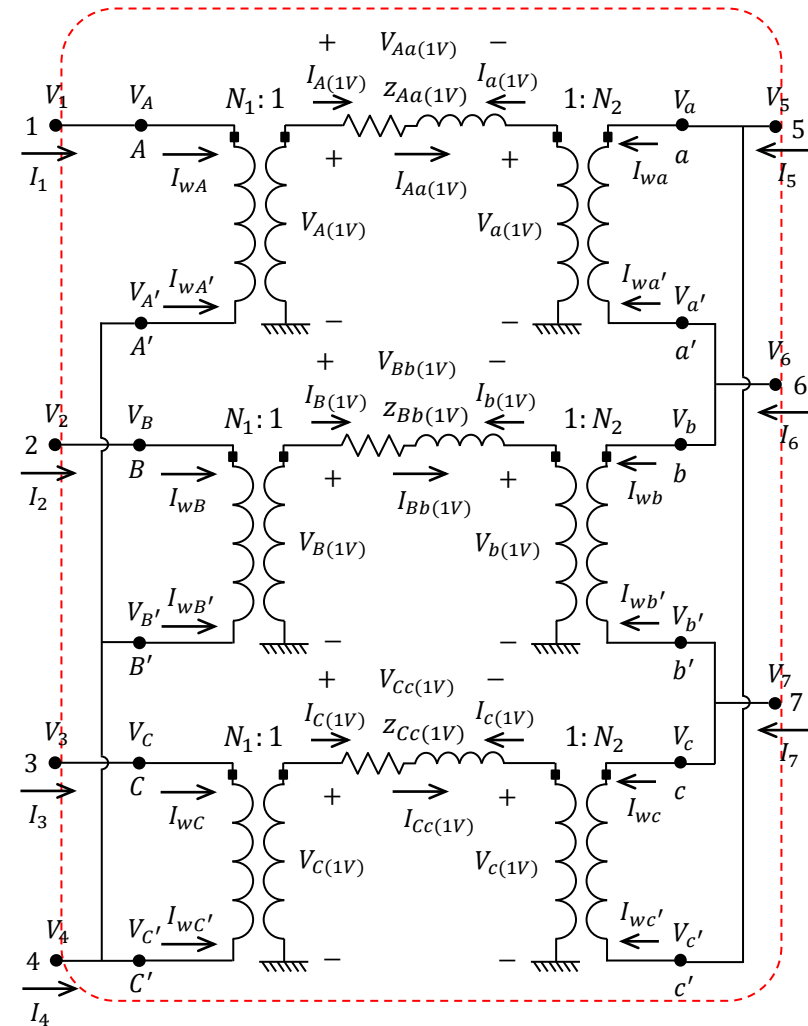
$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \\ I_7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} I_{wA} \\ I_{wA'} \\ I_{wa} \\ I_{wa'} \\ I_{wB} \\ I_{wB'} \\ I_{wb} \\ I_{wb'} \\ I_{wC} \\ I_{wC'} \\ I_{wc} \\ I_{wc'} \end{bmatrix}$$



$$I_{prim} = AI_w, \text{ where, } I_{prim} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \\ I_7 \end{bmatrix}, A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Modeling A Transformer in OpenDSS

4. Three-phase Ungrounded Wye-Delta Step-down Transformer

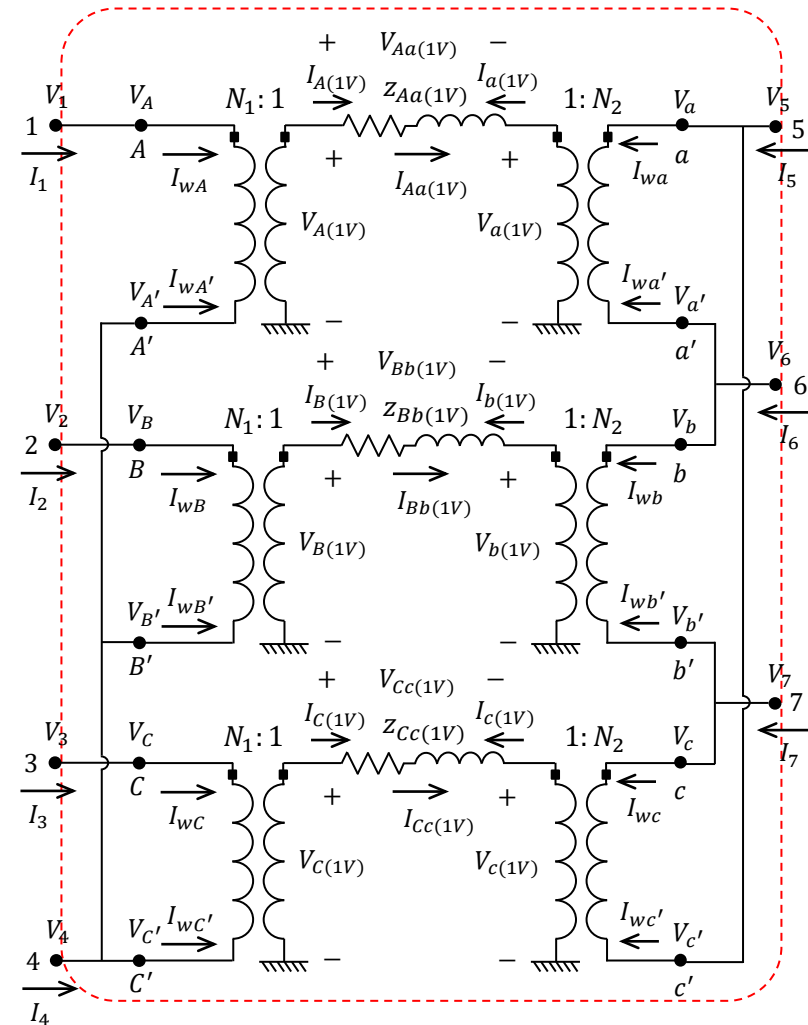


$$\begin{aligned}
 & \textcircled{2} \left\{ \begin{aligned} V_A &= V_1 \\ V_{A'} &= V_4 \\ V_a &= V_5 \\ V_{a'} &= V_6 \\ V_B &= V_2 \\ V_{B'} &= V_4 \\ V_b &= V_6 \\ V_{b'} &= V_7 \\ V_C &= V_3 \\ V_{C'} &= V_4 \\ V_c &= V_7 \\ V_{c'} &= V_5 \end{aligned} \right. \Rightarrow \begin{bmatrix} V_A \\ V_{A'} \\ V_a \\ V_{a'} \\ V_B \\ V_{B'} \\ V_b \\ V_{b'} \\ V_C \\ V_{C'} \\ V_c \\ V_{c'} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \end{bmatrix}
 \end{aligned}$$

$$\Rightarrow \mathbf{V}_w = \mathbf{A}^T \mathbf{V}_{prim}, \text{ where } \mathbf{V}_{prim} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \end{bmatrix}$$

Modeling A Transformer in OpenDSS

4. Three-phase Ungrounded Wye-Delta Step-down Transformer



Let $I_{prim} = Y_{prim} V_{prim}$. How to obtain Y_{prim} ?

$$\left. \begin{aligned} I_{prim} &= Y_{prim} V_{prim} \\ I_{prim} &= A I_w \end{aligned} \right\}$$

$$\Rightarrow \left. \begin{aligned} A I_w &= Y_{prim} V_{prim} \\ V_w &= A^T V_{prim} \end{aligned} \right\}$$

$$\Rightarrow \left. \begin{aligned} A I_w &= Y_{prim} (A^T)^{-1} V_w \\ I_w &= Y_w V_w \end{aligned} \right\}$$

$$\Rightarrow \left. \begin{aligned} A Y_w V_w &= Y_{prim} (A^T)^{-1} V_w \\ Y_w &= N B Y_{sc(1V)} B^T N^T \end{aligned} \right\}$$

$$\Rightarrow Y_{prim} = A N B Y_{sc(1V)} B^T N^T A^T$$

Modeling A Transformer in OpenDSS

4. Three-phase Ungrounded Wye-Delta Step-down Transformer

An example:

// Define a three-phase transformer.

New Transformer.xfmr phases=3 windings=2 xhl=5

~ wdg=1 bus=K.1.2.3.4 conn=wye kV=115 kva=500 %r=1

~ wdg=2 bus=M.1.2.3 conn=delta kV=4.16 kva=500 %r=1

Transformer name

Number of
phases

Number of
windings

Percent reactance
high-to-low

Specify which winding
will be edited

Winding
connection

Rated voltage

Base kVA
rating

Percent
resistance

Computing Y_{prim} using Matlab:

$$Y_{Aa(1V)} = Y_{Bb(1V)} = Y_{Cc(1V)} = \frac{1}{[r(pu) + jx(pu)] * \frac{1^2}{S_t/3}} = \frac{500000}{3 * (0.02 + j0.05)} = 1149425.2873 - j2873563.2184 S$$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix}$$

$$N^T = \begin{bmatrix} \frac{1}{115000/\sqrt{3}} & -\frac{1}{115000/\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4160} & -\frac{1}{4160} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{115000/\sqrt{3}} & -\frac{1}{115000/\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4160} & -\frac{1}{4160} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{115000/\sqrt{3}} & -\frac{1}{115000/\sqrt{3}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4160} & -\frac{1}{4160} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4160} & -\frac{1}{4160} \end{bmatrix}$$

36

Modeling A Transformer in OpenDSS

4. Three-phase Ungrounded Wye-Delta Step-down Transformer

$$\mathbf{Y}_{sc(1V)} = \begin{bmatrix} y_{Aa(1V)} & 0 & 0 \\ 0 & y_{Bb(1V)} & 0 \\ 0 & 0 & y_{Cc(1V)} \end{bmatrix} = \begin{bmatrix} 1149425.2873 - j2873563.2184 & 0 & 0 \\ 0 & 1149425.2873 - j2873563.2184 & 0 \\ 0 & 0 & 1149425.2873 - j2873563.2184 \end{bmatrix}$$

Calculate $\mathbf{Y}_{prim} = \mathbf{ANBY}_{sc(1V)}\mathbf{B}^T\mathbf{N}^T\mathbf{A}^T =$

$$\begin{bmatrix} 0.000260739 - j0.00065184 & 0 + j0 & 0 + j0 & -0.000260739 + j0.00065184 & -0.00416150 + j0.01040375 & 0.00416150 - j0.01040375 & 0 + j0 \\ 0 + j0 & 0.000260739 - j0.00065184 & 0 + j0 & -0.000260739 + j0.00065184 & 0 + j0 & -0.00416150 + j0.01040375 & 0.00416150 - j0.01040375 \\ 0 + j0 & 0 + j0 & 0.000260739 - j0.00065184 & -0.000260739 + j0.00065184 & 0.00416150 - j0.01040375 & 0 + j0 & -0.00416150 + j0.01040375 \\ -0.000260739 + j0.00065184 & -0.000260739 + j0.00065184 & -0.000260739 + j0.00065184 & 0.00078221 - j0.00195554 & 0 + j0 & 0.13283853 - j0.33209634 & -0.06641926 + j0.16604817 \\ -0.00416150 + j0.01040375 & -0.00416150 + j0.01040375 & 0.00416150 - j0.01040375 & 0 + j0 & 0 + j0 & -0.06641926 + j0.16604817 & 0.13283853 - j0.33209634 \\ 0.00416150 - j0.01040375 & 0.00416150 - j0.01040375 & -0.00416150 + j0.01040375 & 0 + j0 & 0 + j0 & -0.06641926 + j0.16604817 & -0.06641926 + j0.16604817 \\ 0 + j0 & 0.00416150 - j0.01040375 & -0.00416150 + j0.01040375 & 0 + j0 & 0 + j0 & -0.06641926 + j0.16604817 & 0.13283853 - j0.33209634 \end{bmatrix}$$

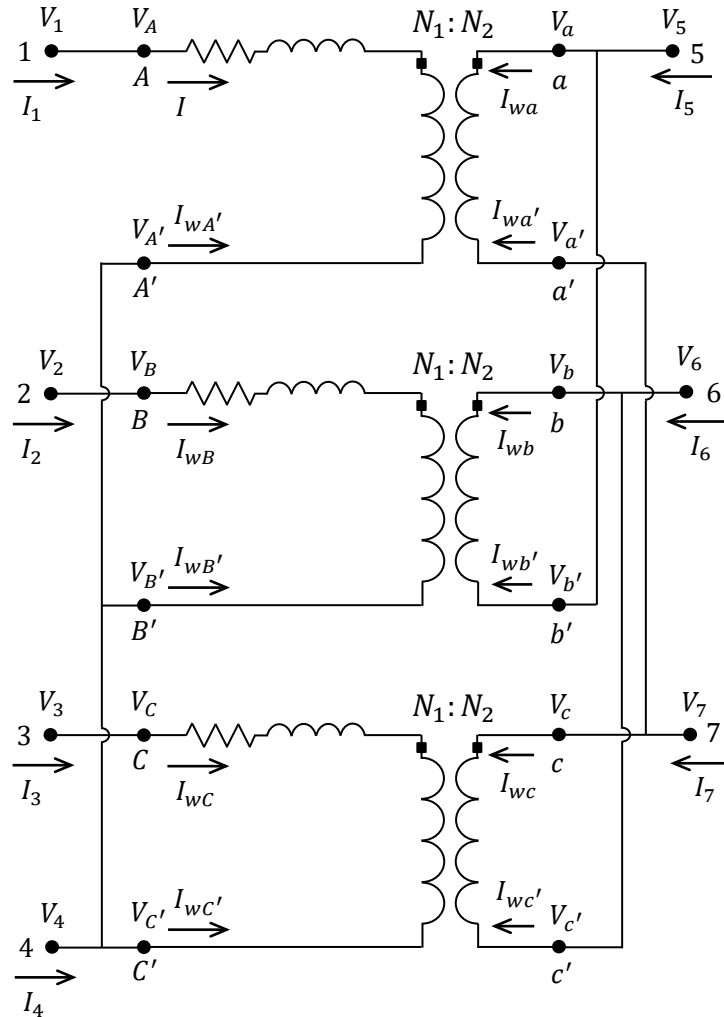
Exported $\mathbf{Y}_{prim}$ from OpenDSS:

	G						B									
1	0.000261	-0.00065	0	0	0	0	-0.00026	0.000652	-0.00416	0.010404	0.004162	-0.0104	0	0	0	0
2	0	0	0.000261	-0.00065	0	0	-0.00026	0.000652	0	0	-0.00416	0.010404	0.004162	-0.0104	0	0
3	0	0	0	0	0.000261	-0.00065	-0.00026	0.000652	0.004162	-0.0104	0	0	-0.00416	0.010404	0	0
4	-0.00026	0.000652	-0.00026	0.000652	-0.00026	0.000652	0.000782	-0.00196	0	0	0	0	0	0	0	0
5	-0.00416	0.010404	0	0	0.004162	-0.0104	0	0	0.132839	-0.3321	-0.06642	0.166048	-0.06642	0.166048	0	0
6	0.004162	-0.0104	-0.00416	0.010404	0	0	0	0	-0.06642	0.166048	0.132839	-0.3321	-0.06642	0.166048	0	0
7	0	0	0.004162	-0.0104	-0.00416	0.010404	0	0	-0.06642	0.166048	-0.06642	0.166048	0.132839	-0.3321	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

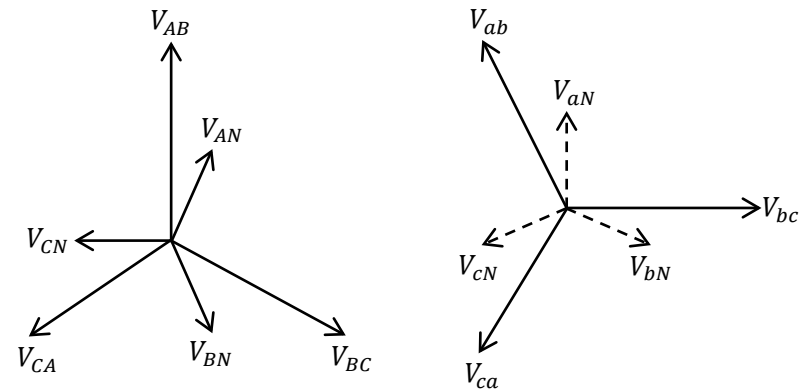
One row and one column of zeros are added to the delta-connected winding to make sure that the primary and second windings have the same size of admittance matrix.

Modeling A Transformer in OpenDSS

5. Three-phase Ungrounded Wye-Delta Step-up Transformer



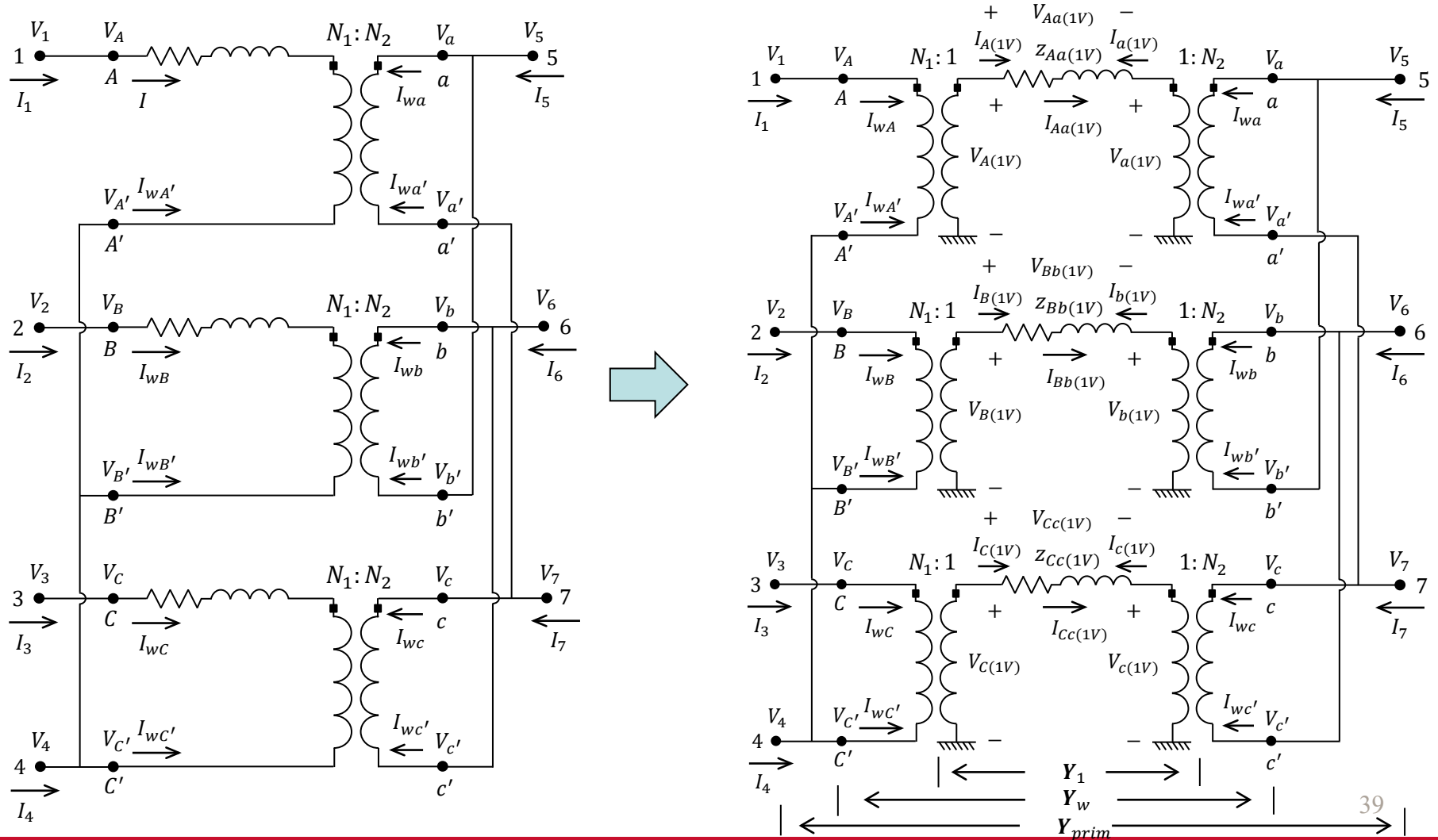
- The HV side always comes before the LV side, regardless of which is the primary winding.
- HV winding is taken as a reference.
- Phase rotation is always counterclockwise.
- 1 = 30° , 2 = 60° , 3 = 90° , 6 = 180° and 12 = 0° or 360° .



Dyn1

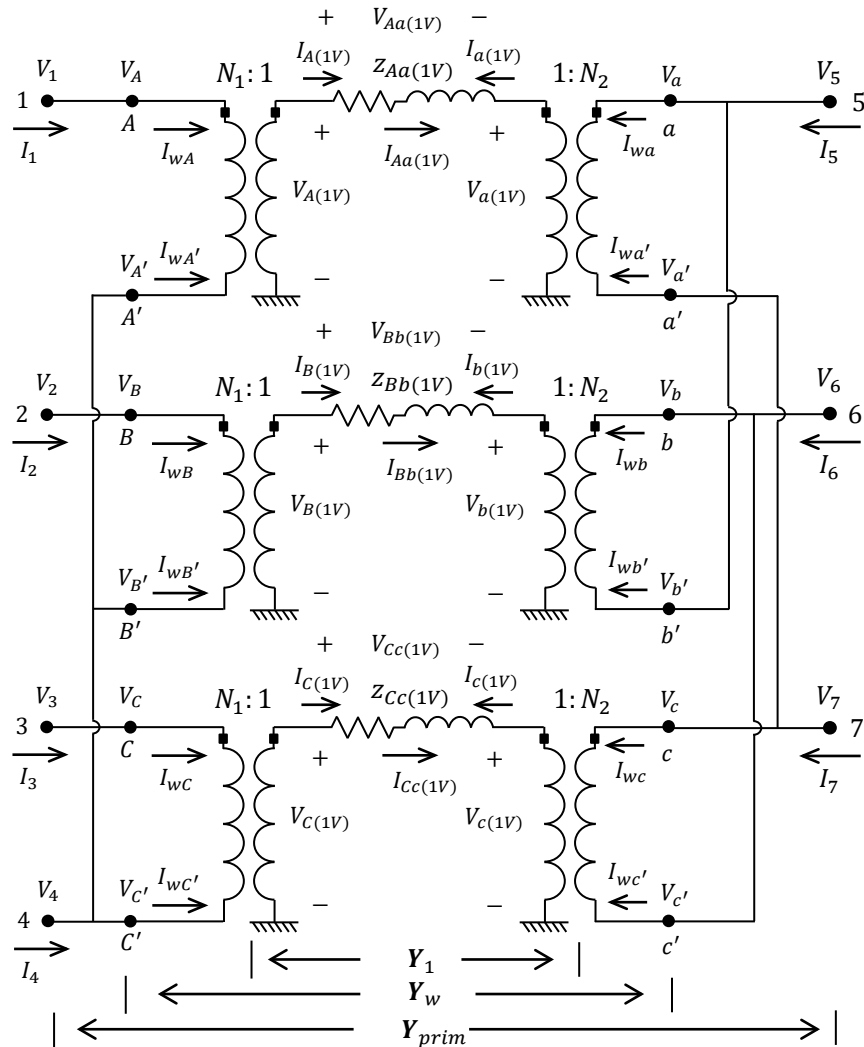
Modeling A Transformer in OpenDSS

5. Three-phase Ungrounded Wye-Delta Step-up Transformer



Modeling A Transformer in OpenDSS

5. Three-phase Ungrounded Wye-Delta Step-up Transformer



Note:

Compared to the 3-phase ungrounded wye-delta step-down transformer, the equation, $\mathbf{Y}_{prim} = \mathbf{ANB}y_{sc(1V)}\mathbf{B}^T\mathbf{N}^T\mathbf{A}^T$, also applies to a 3-phase ungrounded wye-delta step-up transformer. The only difference exists in the formation of matrix $\mathbf{A}$ due to a different connection of delta winding, and the values of N_1 and N_2 . Specifically, according to the figure on the left, we have

$$\begin{cases} I_1 = I_{wA} \\ I_2 = I_{wB} \\ I_3 = I_{wC} \\ I_4 = I_{wA'} + I_{wB'} + I_{wC'} \\ I_5 = I_{wa} + I_{wb'} \\ I_6 = I_{wb} + I_{wc'} \\ I_7 = I_{wc} + I_{wa'} \end{cases} \Rightarrow \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \\ I_7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} I_{wA} \\ I_{wA'} \\ I_{wa} \\ I_{wa'} \\ I_{wB} \\ I_{wB'} \\ I_{wb} \\ I_{wb'} \\ I_{wC} \\ I_{wC'} \\ I_{wc} \\ I_{wc'} \end{bmatrix}$$

$$\Rightarrow \mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Modeling A Transformer in OpenDSS

5. Three-phase Ungrounded Wye-Delta Step-up Transformer

An example:

// Define a three-phase transformer.

New Transformer.xfmr phases=3 windings=2 xhl=5

~ wdg=1 bus=K.1.2.3.4 conn=weye kV=4.16 kva=500 %r=1

~ wdg=2 bus=M.1.2.3 conn=delta kV=115 kva=500 %r=1

Specify which winding
will be edited

Transformer name

Number of
phases

Number of
windings

Percent reactance
high-to-low

Winding
connection

Rated voltage

Base kVA
rating

Percent
resistance

Computing Y_{prim} using Matlab:

$$Y_{Aa(1V)} = Y_{Bb(1V)} = Y_{Cc(1V)} = \frac{1}{[r(pu) + jx(pu)] * \frac{1^2}{S_t/3}} = \frac{500000}{3 * (0.02 + j0.05)} = 1149425.2873 - j2873563.2184$$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix}$$

$$N^T = \begin{bmatrix} \frac{1}{4160/\sqrt{3}} & -\frac{1}{4160/\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{115000} & -\frac{1}{115000} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{4160/\sqrt{3}} & -\frac{1}{4160/\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{115000} & -\frac{1}{115000} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4160/\sqrt{3}} & -\frac{1}{4160/\sqrt{3}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{115000} & -\frac{1}{115000} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{115000} & -\frac{1}{115000} \end{bmatrix}$$

41

Modeling A Transformer in OpenDSS

5. Three-phase Ungrounded Wye-Delta Step-up Transformer

$$\mathbf{Y}_{sc(1V)} = \begin{bmatrix} y_{Aa(1V)} & 0 & 0 \\ 0 & y_{Bb(1V)} & 0 \\ 0 & 0 & y_{Cc(1V)} \end{bmatrix} = \begin{bmatrix} 1149425.2873 - j2873563.2184 & 0 & 0 \\ 0 & 1149425.2873 - j2873563.2184 & 0 \\ 0 & 0 & 1149425.2873 - j2873563.2184 \end{bmatrix}$$

Calculate $\mathbf{Y}_{prim} = \mathbf{ANBY}_{sc(1V)}\mathbf{B}^T\mathbf{N}^T\mathbf{A}^T =$

$$\begin{bmatrix} 0.19925780 - j0.49814451 & 0 + j0 & 0 + j0 & -0.19925780 + j0.49814451 & -0.00416150 + j0.01040375 & 0 + j0 & 0.00416150 - j0.01040375 \\ 0 + j0 & 0.19925780 - j0.49814451 & 0 + j0 & -0.19925780 + j0.49814451 & 0.00416150 - j0.01040375 & -0.00416150 + j0.01040375 & 0 + j0 \\ 0 + j0 & 0 + j0 & 0.19925780 - j0.49814451 & -0.19925780 + j0.49814451 & 0 + j0 & 0.00416150 - j0.01040375 & -0.00416150 + j0.01040375 \\ -0.19925780 + j0.49814451 & -0.19925780 + j0.49814451 & -0.19925780 + j0.49814451 & 0.59777341 - j1.49443353 & 0 + j0 & 0 + j0 & 0 + j0 \\ -0.00416150 + j0.01040375 & 0.00416150 - j0.01040375 & -0.19925780 + j0.49814451 & 0.59777341 - j1.49443353 & 0 + j0 & 0 + j0 & 0 + j0 \\ 0 + j0 & -0.00416150 + j0.01040375 & 0.00416150 - j0.01040375 & 0 + j0 & 0 + j0 & 0 + j0 & 0 + j0 \\ 0.00416150 - j0.01040375 & 0 + j0 & -0.00416150 + j0.01040375 & 0 + j0 & 0 + j0 & 0 + j0 & 0 + j0 \end{bmatrix}$$

Exported $\mathbf{Y}_{prim}$ from OpenDSS:

Exported r_{prim} from OpenDSS.

G

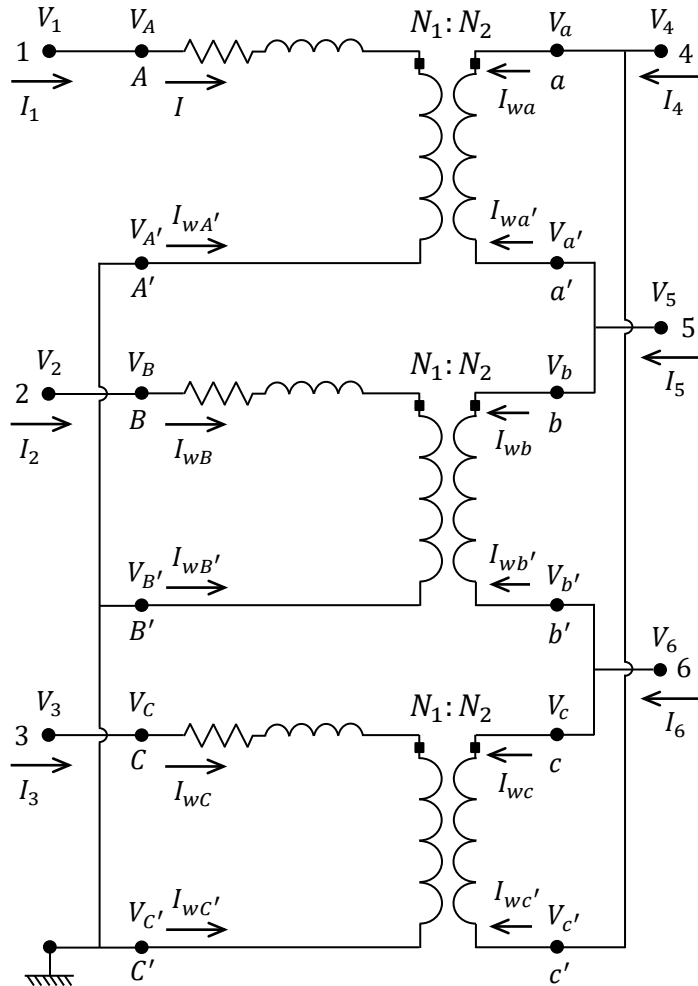
B

1	0.199258	-0.49814	0	0	0	0	-0.19926	0.498145	-0.00416	0.010404	0	0	0.004162	-0.0104	0	0
2	0	0	0.199258	-0.49814	0	0	-0.19926	0.498145	0.004162	-0.0104	-0.00416	0.010404	0	0	0	0
3	0	0	0	0	0.199258	-0.49814	-0.19926	0.498145	0	0	0.004162	-0.0104	-0.00416	0.010404	0	0
4	-0.19926	0.498145	-0.19926	0.498145	-0.19926	0.498145	0.597773	-1.49443	0	0	0	0	0	0	0	0
5	-0.00416	0.010404	0.004162	-0.0104	0	0	0	0	0.000174	-0.00043	-8.69E-05	0.000217	-8.69E-05	0.000217	0	0
6	0	0	-0.00416	0.010404	0.004162	-0.0104	0	0	-8.69E-05	0.000217	0.000174	-0.00043	-8.69E-05	0.000217	0	0
7	0.004162	-0.0104	0	0	-0.00416	0.010404	0	0	-8.69E-05	0.000217	-8.69E-05	0.000217	0.000174	-0.00043	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	1	2	3	4	5	6	7	0								

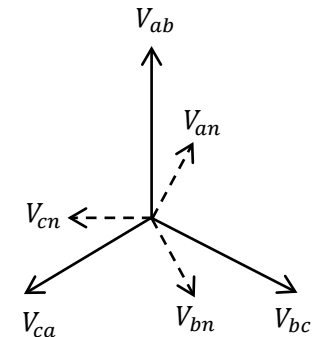
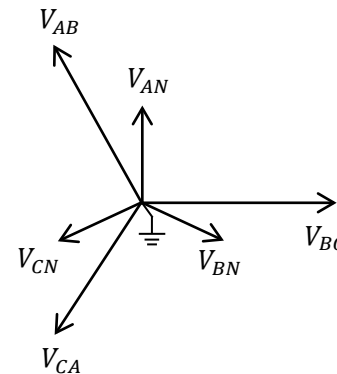
One row and one column of zeros are added to the delta-connected winding to make sure that the primary and second windings have the same size of admittance matrix.

Modeling A Transformer in OpenDSS

6. Three-phase Grounded Wye-Delta Step-down Transformer



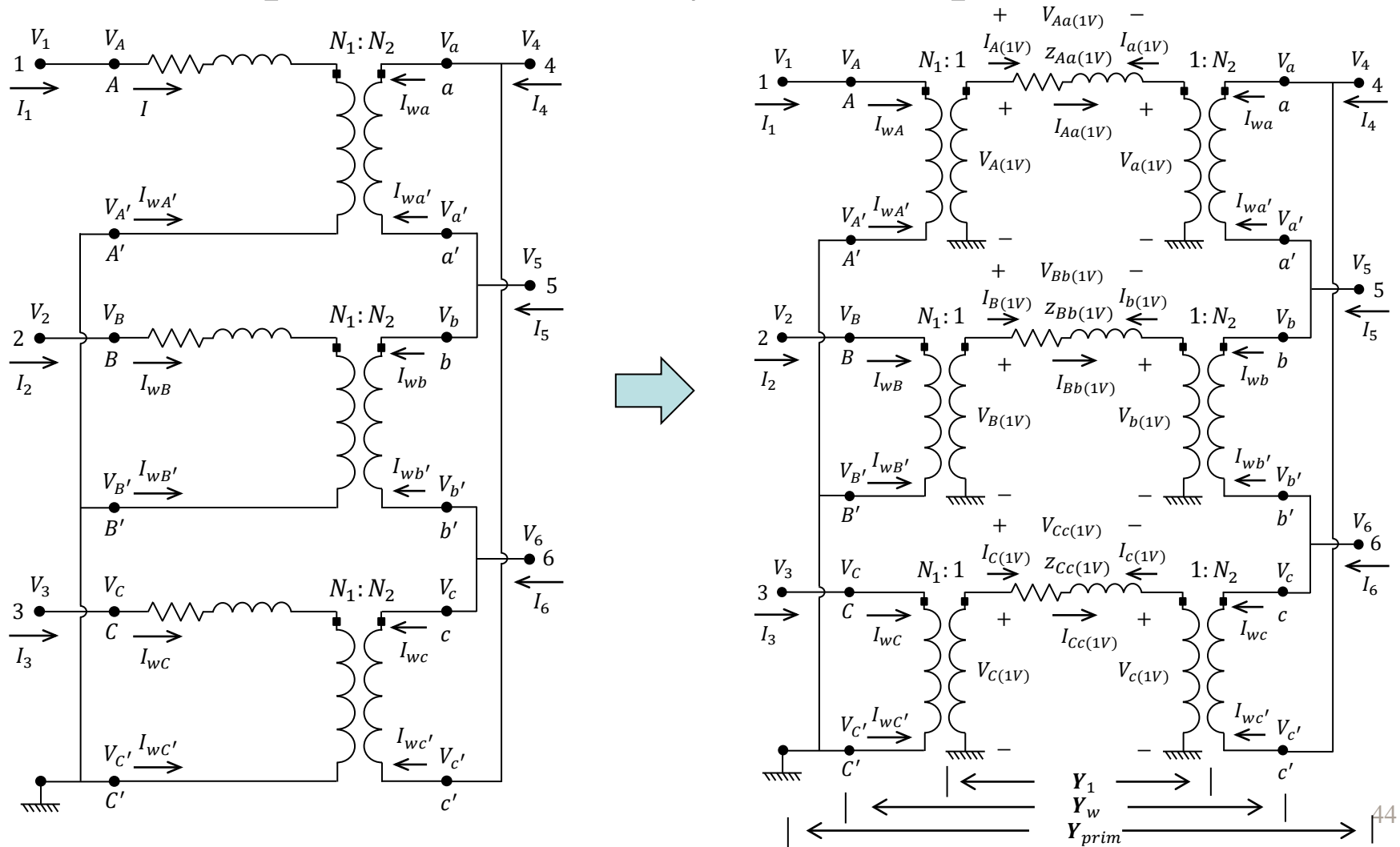
- The HV side always comes before the LV side, regardless of which is the primary winding.
- HV winding is taken as a reference.
- Phase rotation is always counterclockwise.
- $1 = 30^\circ$, $2 = 60^\circ$, $3 = 90^\circ$, $6 = 180^\circ$ and $12 = 0^\circ$ or 360° .



YNd1

Modeling A Transformer in OpenDSS

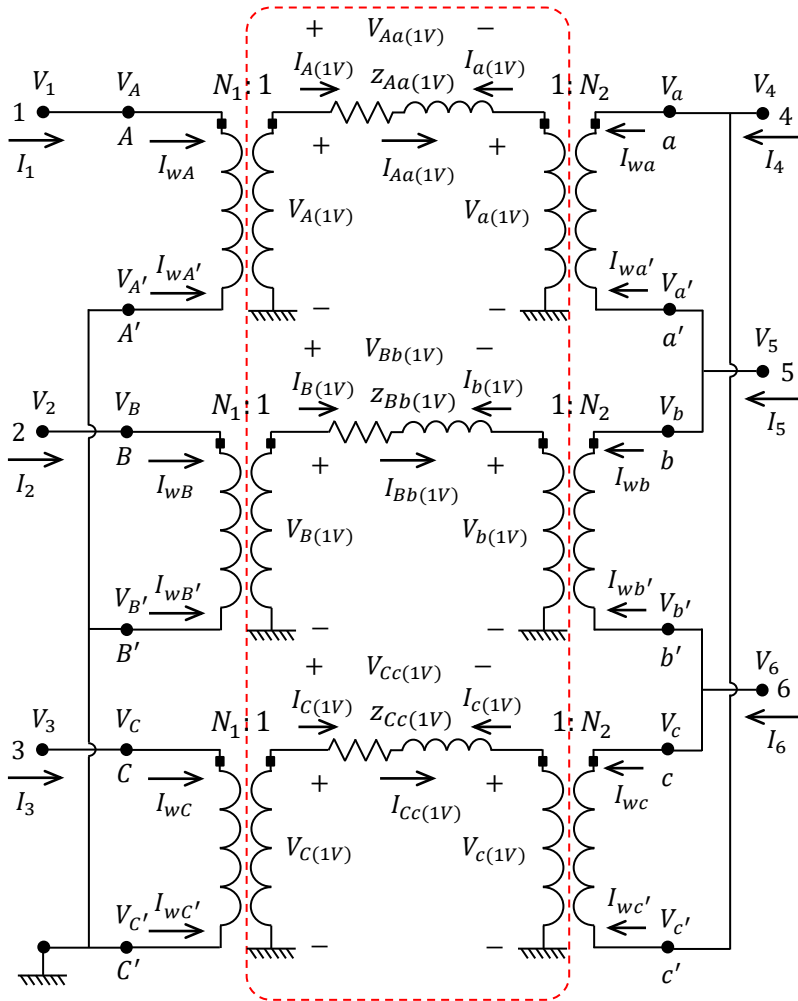
6. Three-phase Grounded Wye-Delta Step-down Transformer



Modeling A Transformer in OpenDSS

6. Three-phase Grounded Wye-Delta Step-down Transformer

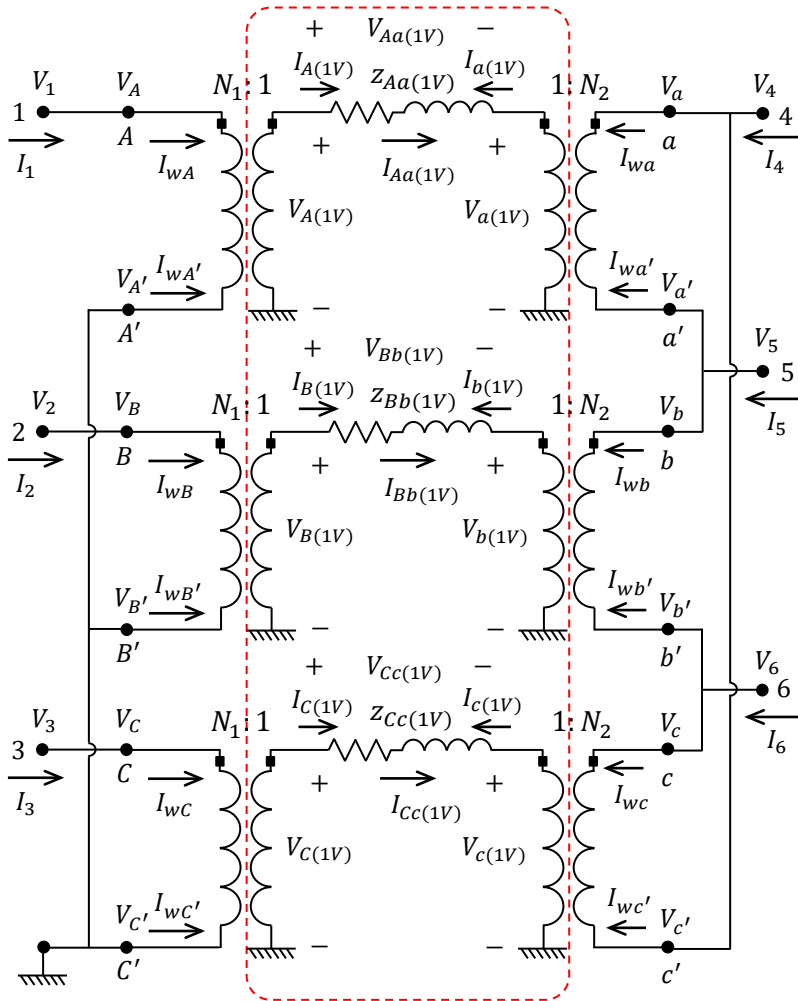
Step 1: Compute the nodal admittance matrix on a one turn or one voltage base.



$$\begin{aligned}
 & \text{① } \begin{cases} I_A(1V) = I_{Aa}(1V) \\ I_a(1V) = -I_{Aa}(1V) \\ I_B(1V) = I_{Bb}(1V) \\ I_b(1V) = -I_{Bb}(1V) \\ I_C(1V) = I_{Cc}(1V) \\ I_c(1V) = -I_{Cc}(1V) \end{cases} \Rightarrow \begin{bmatrix} I_A(1V) \\ I_a(1V) \\ I_B(1V) \\ I_b(1V) \\ I_C(1V) \\ I_c(1V) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} I_{Aa}(1V) \\ I_{Bb}(1V) \\ I_{Cc}(1V) \end{bmatrix} \\
 & \Rightarrow I_1 = B I_{sc(1V)}, \text{ where } I_1 = \begin{bmatrix} I_A(1V) \\ I_a(1V) \\ I_B(1V) \\ I_b(1V) \\ I_C(1V) \\ I_c(1V) \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix}, I_{sc(1V)} = \begin{bmatrix} I_{Aa}(1V) \\ I_{Bb}(1V) \\ I_{Cc}(1V) \end{bmatrix} \\
 & \text{② } \begin{cases} V_{Aa}(1V) = V_A(1V) - V_a(1V) \\ V_{Bb}(1V) = V_B(1V) - V_b(1V) \\ V_{Cc}(1V) = V_C(1V) - V_c(1V) \end{cases} \Rightarrow \begin{bmatrix} V_{Aa}(1V) \\ V_{Bb}(1V) \\ V_{Cc}(1V) \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} V_A(1V) \\ V_a(1V) \\ V_B(1V) \\ V_b(1V) \\ V_C(1V) \\ V_c(1V) \end{bmatrix} \\
 & \Rightarrow V_{sc(1V)} = B^T V_1, \text{ where } V_{sc(1V)} = \begin{bmatrix} V_{Aa}(1V) \\ V_{Bb}(1V) \\ V_{Cc}(1V) \end{bmatrix}, V_1 = \begin{bmatrix} V_A(1V) \\ V_a(1V) \\ V_B(1V) \\ V_b(1V) \\ V_C(1V) \\ V_c(1V) \end{bmatrix} \\
 & \text{③ } \begin{cases} V_{Aa}(1V) = Z_{Aa}(1V) I_{Aa}(1V) \\ V_{Bb}(1V) = Z_{Bb}(1V) I_{Bb}(1V) \\ V_{Cc}(1V) = Z_{Cc}(1V) I_{Cc}(1V) \end{cases} \Rightarrow \begin{bmatrix} V_{Aa}(1V) \\ V_{Bb}(1V) \\ V_{Cc}(1V) \end{bmatrix} = \begin{bmatrix} Z_{Aa}(1V) & 0 & 0 \\ 0 & Z_{Bb}(1V) & 0 \\ 0 & 0 & Z_{Cc}(1V) \end{bmatrix} \begin{bmatrix} I_{Aa}(1V) \\ I_{Bb}(1V) \\ I_{Cc}(1V) \end{bmatrix} \\
 & \Rightarrow V_{sc(1V)} = Z_{sc(1V)} I_{sc(1V)} \Rightarrow Y_{sc(1V)} V_{sc(1V)} = I_{sc(1V)}, \text{ where } Y_{sc(1V)} = (Z_{sc(1V)})^{-1}
 \end{aligned}$$

Modeling A Transformer in OpenDSS

6. Three-phase Grounded Wye-Delta Step-down Transformer



Let $I_1 = Y_1 V_1$, where Y_1 is the admittance matrix on a one turn or one voltage base. What is Y_1 ?

$$\left. \begin{aligned} I_1 &= Y_1 V_1 \\ I_1 &= B I_{sc(1V)} \end{aligned} \right\}$$

$$\Rightarrow \left. \begin{aligned} B I_{sc(1V)} &= Y_1 V_1 \\ V_{sc(1V)} &= B^T V_1 \end{aligned} \right\}$$

$$\Rightarrow \left. \begin{aligned} B I_{sc(1V)} &= Y_1 (B^T)^{-1} V_{sc(1V)} \\ I_{sc(1V)} &= Y_{sc(1V)} V_{sc(1V)} \end{aligned} \right\}$$

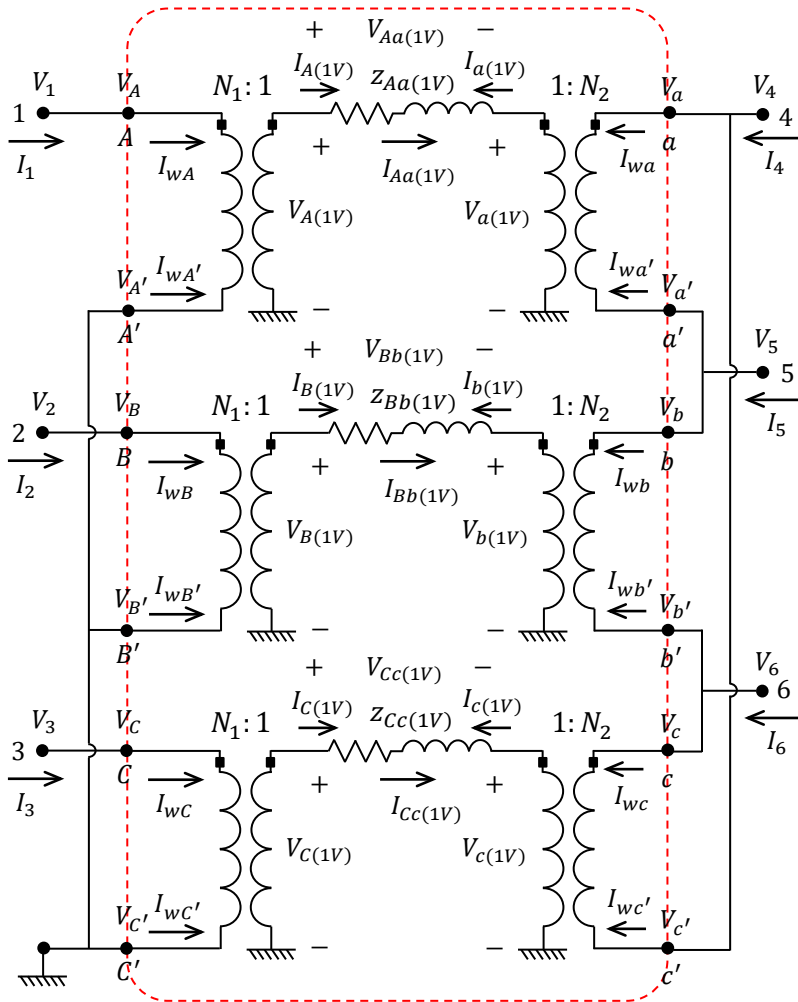
$$\Rightarrow B Y_{sc(1V)} V_{sc(1V)} = Y_1 (B^T)^{-1} V_{sc(1V)}$$

$$\Rightarrow Y_1 = B Y_{sc(1V)} B^T$$

Modeling A Transformer in OpenDSS

6. Three-phase Grounded Wye-Delta Step-down Transformer

Step 2: Consider winding turns ratio.



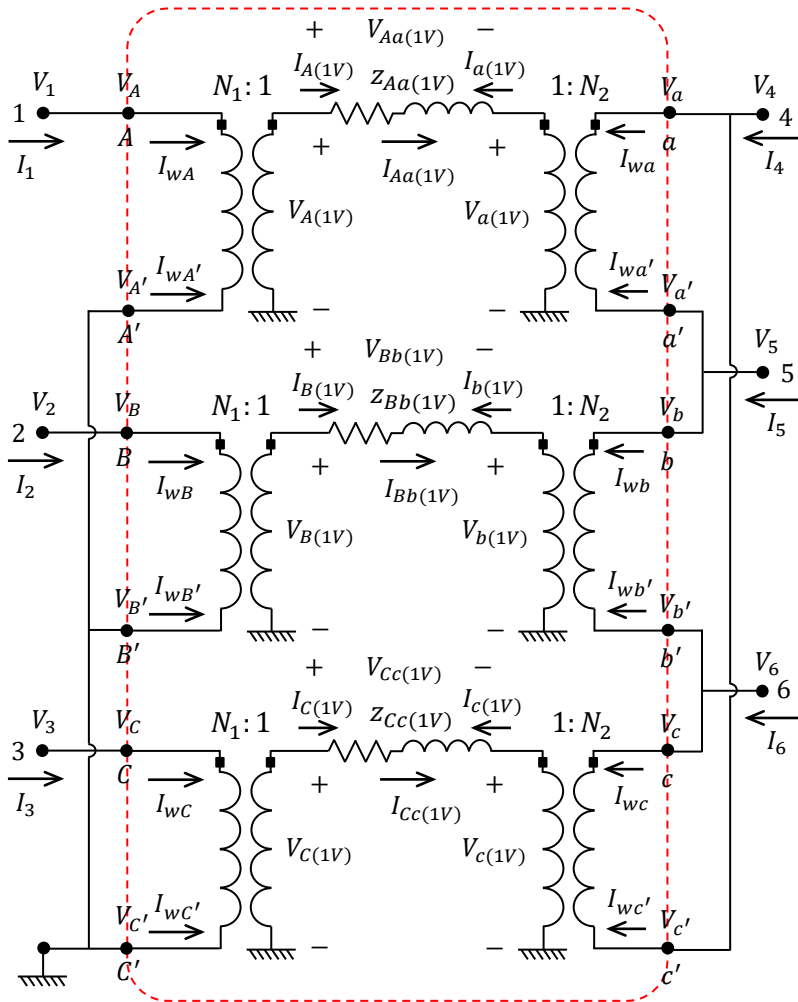
①

$$\begin{cases} I_{wA} = I_{A(1V)}/N_1 \\ I_{wA'} = -I_{A(1V)}/N_1 \\ I_{wa} = I_{a(1V)}/N_2 \\ I_{wa'} = -I_{a(1V)}/N_2 \\ I_{wB} = I_{B(1V)}/N_1 \\ I_{wB'} = -I_{B(1V)}/N_1 \\ I_{wb} = I_{b(1V)}/N_2 \\ I_{wb'} = -I_{b(1V)}/N_2 \\ I_{wC} = I_{C(1V)}/N_1 \\ I_{wC'} = -I_{C(1V)}/N_1 \\ I_{wc} = I_{c(1V)}/N_2 \\ I_{wc'} = -I_{c(1V)}/N_2 \end{cases} \Rightarrow \begin{bmatrix} I_{wA} \\ I_{wA'} \\ I_{wa} \\ I_{wa'} \\ I_{wB} \\ I_{wB'} \\ I_{wb} \\ I_{wb'} \\ I_{wC} \\ I_{wC'} \\ I_{wc} \\ I_{wc'} \end{bmatrix} = \begin{bmatrix} \frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{N_2} & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{N_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{N_1} & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{N_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{N_2} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{N_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{N_1} & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{N_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{N_2} \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{N_2} \end{bmatrix} \begin{bmatrix} I_{A(1V)} \\ I_{a(1V)} \\ I_{B(1V)} \\ I_{b(1V)} \\ I_{C(1V)} \\ I_{c(1V)} \end{bmatrix}$$

$$\Rightarrow I_w = NI_1, \text{ where } I_w = \begin{bmatrix} I_{wA} \\ I_{wA'} \\ I_{wa} \\ I_{wa'} \\ I_{wB} \\ I_{wB'} \\ I_{wb} \\ I_{wb'} \\ I_{wC} \\ I_{wC'} \\ I_{wc} \\ I_{wc'} \end{bmatrix}, N = \begin{bmatrix} \frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{N_2} & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{N_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{N_1} & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{N_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{N_2} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{N_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{N_1} & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{N_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{N_2} \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{N_2} \end{bmatrix}$$

Modeling A Transformer in OpenDSS

6. Three-phase Grounded Wye-Delta Step-down Transformer



2

$$\begin{cases} V_{wA} - V_{wA'} = V_{A(1V)}N_1 \\ V_{wa} - V_{wa'} = V_{a(1V)}N_2 \\ V_{wB} - V_{wB'} = V_{B(1V)}N_1 \\ V_{wb} - V_{wb'} = V_{b(1V)}N_2 \\ V_{wC} - V_{wC'} = V_{C(1V)}N_1 \\ V_{wc} - V_{wc'} = V_{c(1V)}N_2 \end{cases}$$



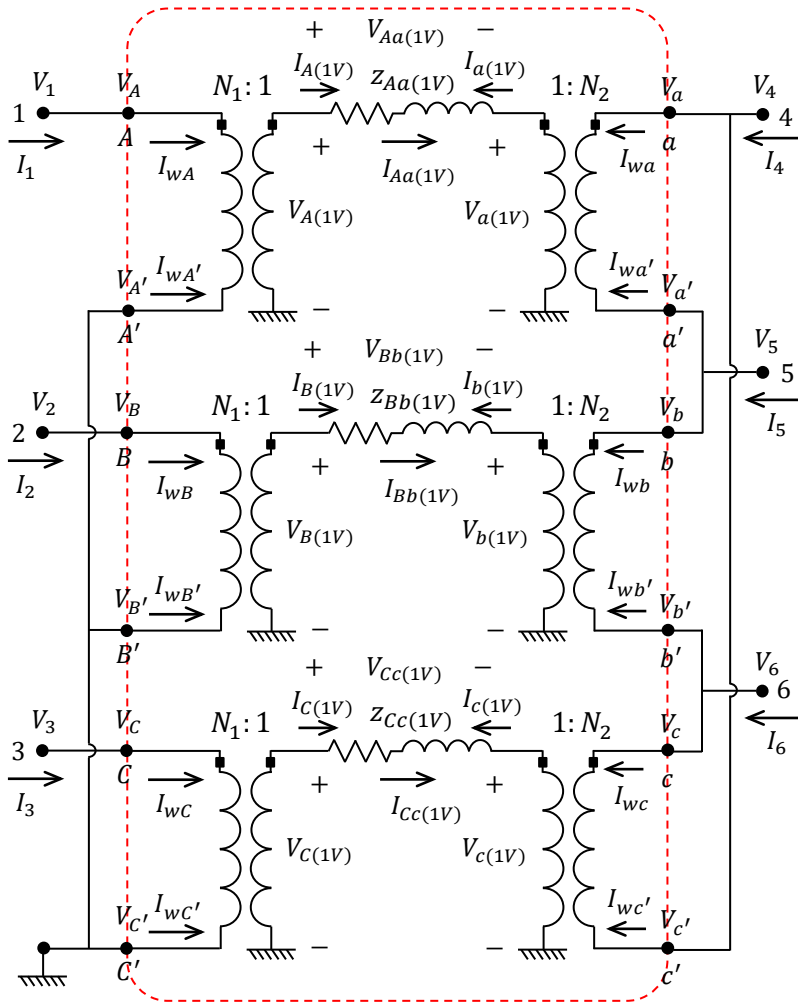
$$\begin{bmatrix} V_{A(1V)} \\ V_{a(1V)} \\ V_{B(1V)} \\ V_{b(1V)} \\ V_{C(1V)} \\ V_{c(1V)} \end{bmatrix} = \begin{bmatrix} \frac{1}{N_1} & -\frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{N_2} & -\frac{1}{N_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{N_1} & -\frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{N_2} & -\frac{1}{N_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{N_1} & -\frac{1}{N_1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{N_2} & -\frac{1}{N_2} \end{bmatrix} \begin{bmatrix} V_{wA} \\ V_{wA'} \\ V_{wa} \\ V_{wa'} \\ V_{wB} \\ V_{wB'} \\ V_{wb} \\ V_{wb'} \\ V_{wC} \\ V_{wC'} \\ V_{wc} \\ V_{wc'} \end{bmatrix}$$



$$V_1 = N^T V_w, \text{ where } V_w = \begin{bmatrix} V_{wA} \\ V_{wA'} \\ V_{wa} \\ V_{wa'} \\ V_{wB} \\ V_{wB'} \\ V_{wb} \\ V_{wb'} \\ V_{wC} \\ V_{wC'} \\ V_{wc} \\ V_{wc'} \end{bmatrix}$$

Modeling A Transformer in OpenDSS

6. Three-phase Grounded Wye-Delta Step-down Transformer



Next, let $I_w = Y_w V_w$. How to obtain Y_w ?

$$\left. \begin{aligned} I_w &= Y_w V_w \\ I_w &= N I_1 \\ V_1 &= N^T V_w \end{aligned} \right\}$$

$$\Rightarrow \left. \begin{aligned} N I_1 &= Y_w (N^T)^{-1} V_1 \\ I_1 &= Y_1 V_1 \end{aligned} \right\}$$

$$\Rightarrow N Y_1 V_1 = Y_w (N^T)^{-1} V_1$$

$$\Rightarrow \left. \begin{aligned} Y_w &= N Y_1 N^T \\ Y_1 &= B Y_{sc(1V)} B^T \end{aligned} \right\}$$

$$\Rightarrow Y_w = N B Y_{sc(1V)} B^T N^T$$

Modeling A Transformer in OpenDSS

6. Three-phase Grounded Wye-Delta Step-down Transformer

Step 3: Consider winding connections

①

$$\begin{cases} I_1 = I_{wA} \\ I_2 = I_{wB} \\ I_3 = I_{wC} \\ I_4 = I_{wa} + I_{wc'} \\ I_5 = I_{wa'} + I_{wb} \\ I_6 = I_{wb'} + I_{wc} \end{cases}$$

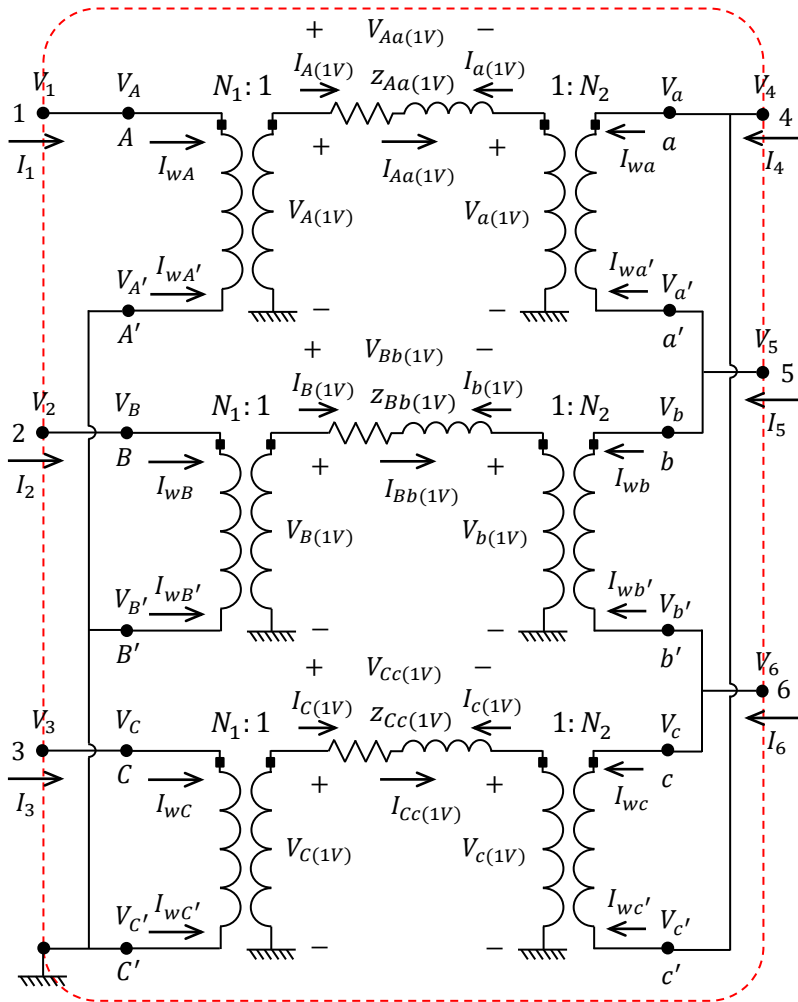


$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} I_{wA} \\ I_{wA'} \\ I_{wa} \\ I_{wa'} \\ I_{wB} \\ I_{wB'} \\ I_{wb} \\ I_{wb'} \\ I_{wC} \\ I_{wC'} \\ I_{wc} \\ I_{wc'} \end{bmatrix}$$

$$\Rightarrow I_{prim} = AI_w, \text{ where, } I_{prim} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix}, A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Modeling A Transformer in OpenDSS

6. Three-phase Grounded Wye-Delta Step-down Transformer

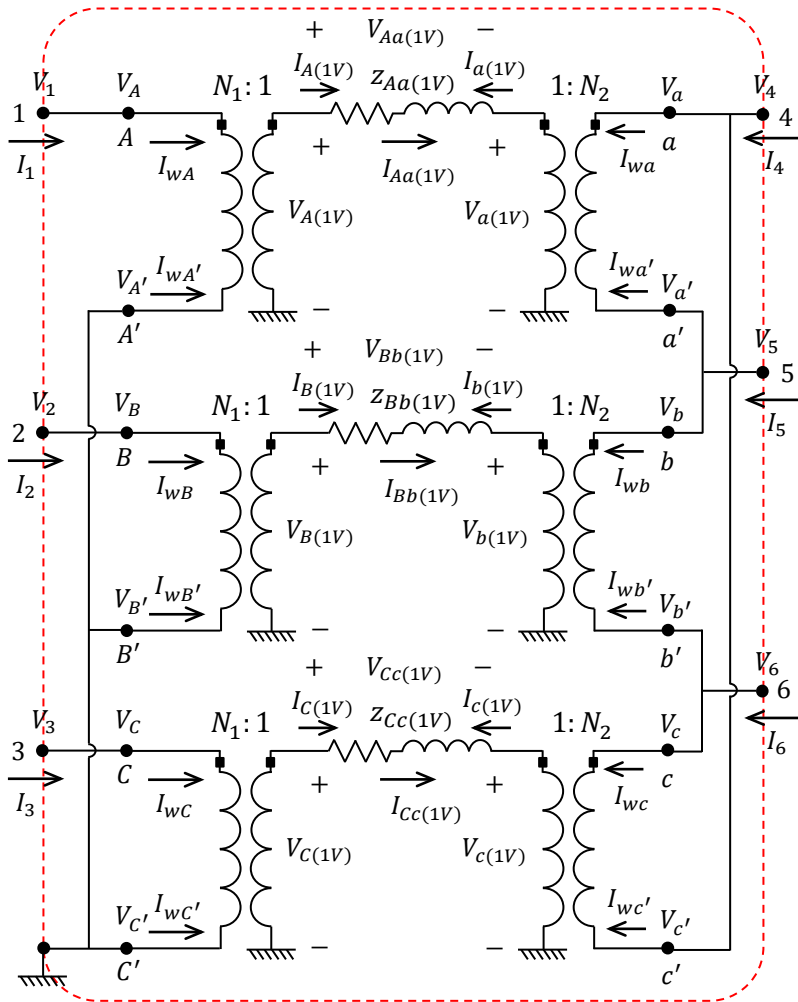


$$\textcircled{2} \left\{ \begin{array}{l} V_A = V_1 \\ V_{A'} = 0 \\ V_a = V_4 \\ V_{a'} = V_5 \\ V_B = V_2 \\ V_{B'} = 0 \\ V_b = V_5 \\ V_{b'} = V_6 \\ V_C = V_3 \\ V_{C'} = 0 \\ V_c = V_6 \\ V_{c'} = V_4 \end{array} \right. \Rightarrow \begin{bmatrix} V_A \\ V_{A'} \\ V_a \\ V_{a'} \\ V_B \\ V_{B'} \\ V_b \\ V_{b'} \\ V_C \\ V_{C'} \\ V_c \\ V_{c'} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix}$$

$$\Rightarrow \mathbf{V}_w = \mathbf{A}^T \mathbf{V}_{prim}, \text{ where } \mathbf{V}_{prim} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix}.$$

Modeling A Transformer in OpenDSS

6. Three-phase Grounded Wye-Delta Step-down Transformer



Let $I_{prim} = Y_{prim} V_{prim}$. How to obtain Y_{prim} ?

$$\left. \begin{aligned} I_{prim} &= Y_{prim} V_{prim} \\ I_{prim} &= A I_w \end{aligned} \right\}$$

$$\Rightarrow \left. \begin{aligned} A I_w &= Y_{prim} V_{prim} \\ V_w &= A^T V_{prim} \end{aligned} \right\}$$

$$\Rightarrow \left. \begin{aligned} A I_w &= Y_{prim} (A^T)^{-1} V_w \\ I_w &= Y_w V_w \end{aligned} \right\}$$

$$\Rightarrow \left. \begin{aligned} A Y_w V_w &= Y_{prim} (A^T)^{-1} V_w \\ Y_w &= N B Y_{sc(1V)} B^T N^T \end{aligned} \right\}$$

$$\Rightarrow Y_{prim} = A N B Y_{sc(1V)} B^T N^T A^T$$

Modeling A Transformer in OpenDSS

6. Three-phase Grounded Wye-Delta Step-down Transformer

An example:

```
// Define a three-phase transformer.
New Transformer.xfm phases=3 windings=2 xhl=5
~ wdg=1 bus=K.1.2.3.0 conn=weye kV=115 kva=500 %r=1
~ wdg=2 bus=M.1.2.3 conn=delta kV=4.16 kva=500 %r=1
```

Specify which winding will be edited

Winding connection

Rated voltage

Base kVA rating

Percent resistance

Computing Y_{prim} using Matlab:

$$Y_{Aa(1V)} = Y_{Bb(1V)} = Y_{Cc(1V)} = \frac{1}{[r(pu) + jx(pu)] * \frac{1^2}{S_t/3}} = \frac{500000}{3 * (0.02 + j0.05)} = 1149425.2873 - j2873563.2184 S$$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix}$$

$$N^T = \begin{bmatrix} \frac{1}{115000/\sqrt{3}} & -\frac{1}{115000/\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4160} & -\frac{1}{4160} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{115000/\sqrt{3}} & -\frac{1}{115000/\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4160} & -\frac{1}{4160} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{115000/\sqrt{3}} & -\frac{1}{115000/\sqrt{3}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4160} & -\frac{1}{4160} \end{bmatrix}$$

53

Modeling A Transformer in OpenDSS

6. Three-phase Grounded Wye-Delta Step-down Transformer

$$\mathbf{Y}_{sc(1V)} = \begin{bmatrix} y_{Aa(1V)} & 0 & 0 \\ 0 & y_{Bb(1V)} & 0 \\ 0 & 0 & y_{Cc(1V)} \end{bmatrix} = \begin{bmatrix} 1149425.2873 - j2873563.2184 & 0 & 0 \\ 0 & 1149425.2873 - j2873563.2184 & 0 \\ 0 & 0 & 1149425.2873 - j2873563.2184 \end{bmatrix}$$

Calculate $\mathbf{Y}_{prim} = \mathbf{A}\mathbf{N}\mathbf{B}\mathbf{Y}_{sc(1V)}\mathbf{B}^T\mathbf{N}^T\mathbf{A}^T =$

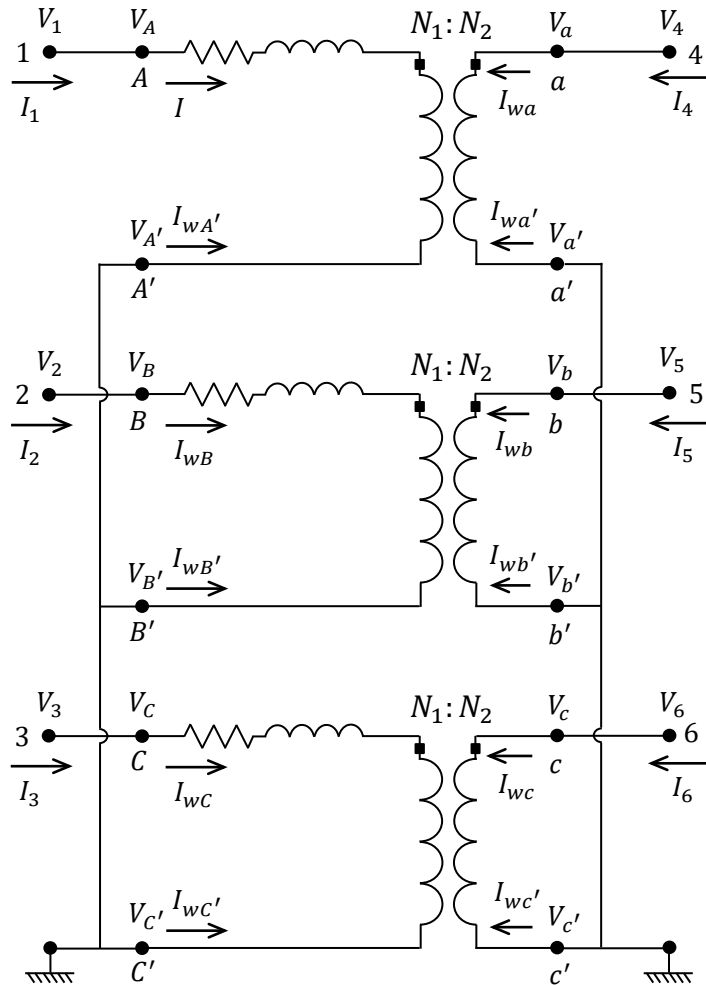
$$\begin{bmatrix} 0.000260739 - j0.00065184 & 0 + j0 & 0 + j0 & -0.00416150 + j0.01040375 & 0.00416150 - j0.01040375 & 0 + j0 \\ 0 + j0 & 0.000260739 - j0.00065184 & 0 + j0 & 0 + j0 & -0.00416150 + j0.01040375 & 0.00416150 - j0.01040375 \\ 0 + j0 & 0 + j0 & 0.000260739 - j0.00065184 & 0.00416150 - j0.01040375 & 0 + j0 & -0.00416150 + j0.01040375 \\ -0.00416150 + j0.01040375 & 0 + j0 & 0.00416150 - j0.01040375 & 0.13283853 - j0.33209634 & -0.06641926 + j0.16604817 & -0.06641926 + j0.16604817 \\ 0.00416150 - j0.01040375 & -0.00416150 + j0.01040375 & 0.00416150 - j0.01040375 & -0.06641926 + j0.16604817 & 0.13283853 - j0.33209634 & -0.06641926 + j0.16604817 \\ 0 + j0 & 0.00416150 - j0.01040375 & -0.00416150 + j0.01040375 & -0.06641926 + j0.16604817 & -0.06641926 + j0.16604817 & 0.13283853 - j0.33209634 \end{bmatrix}$$

Exported $\mathbf{Y}_{prim}$ from OpenDSS:

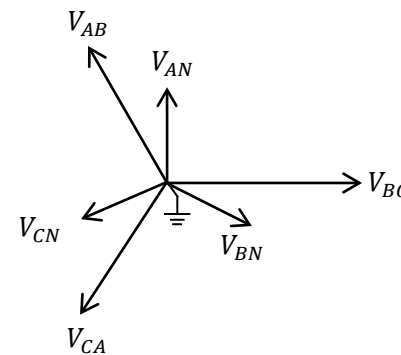
	G			B								
	↓			↓								
1	0.000261	-0.00065	0	0	0	0	-0.00416	0.010404	0.004162	-0.0104	0	0
2	0	0	0.000261	-0.00065	0	0	0	0	-0.00416	0.010404	0.004162	-0.0104
3	0	0	0	0	0.000261	-0.00065	0.004162	-0.0104	0	0	-0.00416	0.010404
4	-0.00416	0.010404	0	0	0.004162	-0.0104	0.132839	-0.3321	-0.06642	0.166048	-0.06642	0.166048
5	0.004162	-0.0104	-0.00416	0.010404	0	0	-0.06642	0.166048	0.132839	-0.3321	-0.06642	0.166048
6	0	0	0.004162	-0.0104	-0.00416	0.010404	-0.06642	0.166048	-0.06642	0.166048	0.132839	-0.3321
	1	2	3	4	5	6						

Modeling A Transformer in OpenDSS

7. Three-phase Grounded Wye-Grounded Wye Transformer



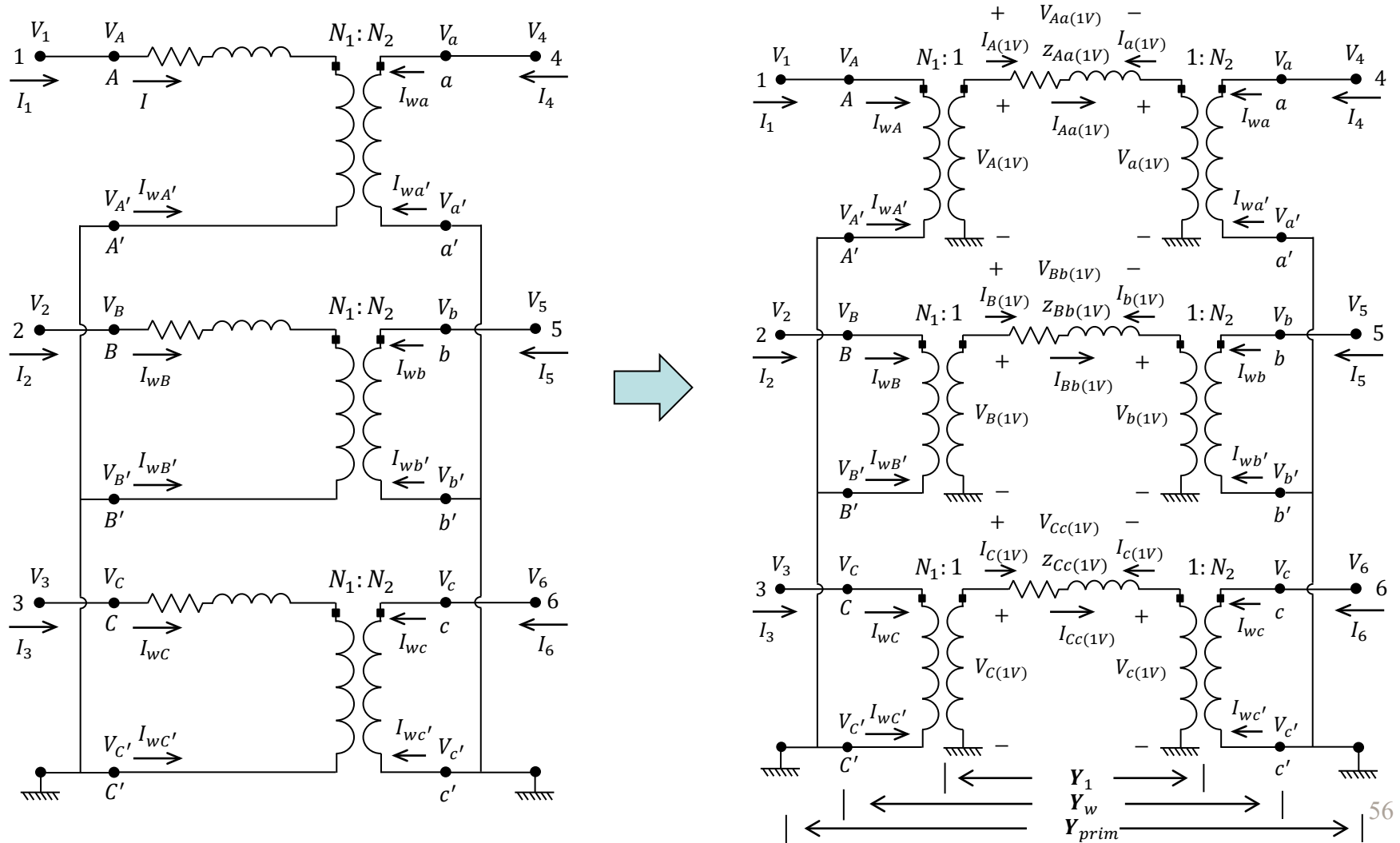
- The HV side always comes before the LV side, regardless of which is the primary winding.
- HV winding is taken as a reference.
- Phase rotation is always counterclockwise.
- $1 = 30^\circ, 2 = 60^\circ, 3 = 90^\circ, 6 = 180^\circ$ and $12 = 0^\circ$ or 360° .



YNyn0

Modeling A Transformer in OpenDSS

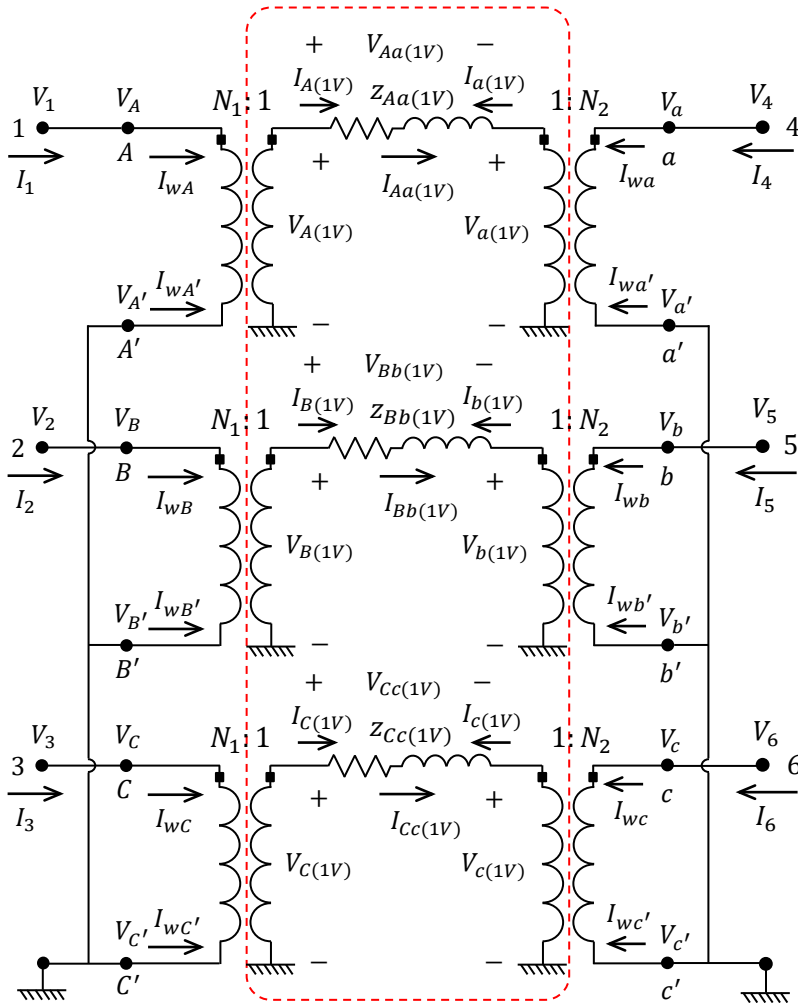
7. Three-phase Grounded Wye-Grounded Wye Transformer



Modeling A Transformer in OpenDSS

7. Three-phase Grounded Wye-Grounded Wye Transformer

Step 1: Compute the nodal admittance matrix on a one turn or one voltage base.



$$\textcircled{1} \quad \begin{cases} I_A(1V) = I_{Aa}(1V) \\ I_a(1V) = -I_{Aa}(1V) \\ I_B(1V) = I_{Bb}(1V) \\ I_b(1V) = -I_{Bb}(1V) \\ I_C(1V) = I_{Cc}(1V) \\ I_c(1V) = -I_{Cc}(1V) \end{cases} \Rightarrow \begin{bmatrix} I_A(1V) \\ I_a(1V) \\ I_B(1V) \\ I_b(1V) \\ I_C(1V) \\ I_c(1V) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} I_{Aa}(1V) \\ I_{Bb}(1V) \\ I_{Cc}(1V) \end{bmatrix}$$

$$\Rightarrow \mathbf{I}_1 = \mathbf{B} \mathbf{I}_{sc(1V)}, \text{ where } \mathbf{I}_1 = \begin{bmatrix} I_A(1V) \\ I_a(1V) \\ I_B(1V) \\ I_b(1V) \\ I_C(1V) \\ I_c(1V) \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix}, \mathbf{I}_{sc(1V)} = \begin{bmatrix} I_{Aa}(1V) \\ I_{Bb}(1V) \\ I_{Cc}(1V) \end{bmatrix}$$

$$\textcircled{2} \quad \begin{cases} V_{Aa}(1V) = V_A(1V) - V_a(1V) \\ V_{Bb}(1V) = V_B(1V) - V_b(1V) \\ V_{Cc}(1V) = V_C(1V) - V_c(1V) \end{cases} \Rightarrow \begin{bmatrix} V_{Aa}(1V) \\ V_{Bb}(1V) \\ V_{Cc}(1V) \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} V_A(1V) \\ V_a(1V) \\ V_B(1V) \\ V_b(1V) \\ V_C(1V) \\ V_c(1V) \end{bmatrix}$$

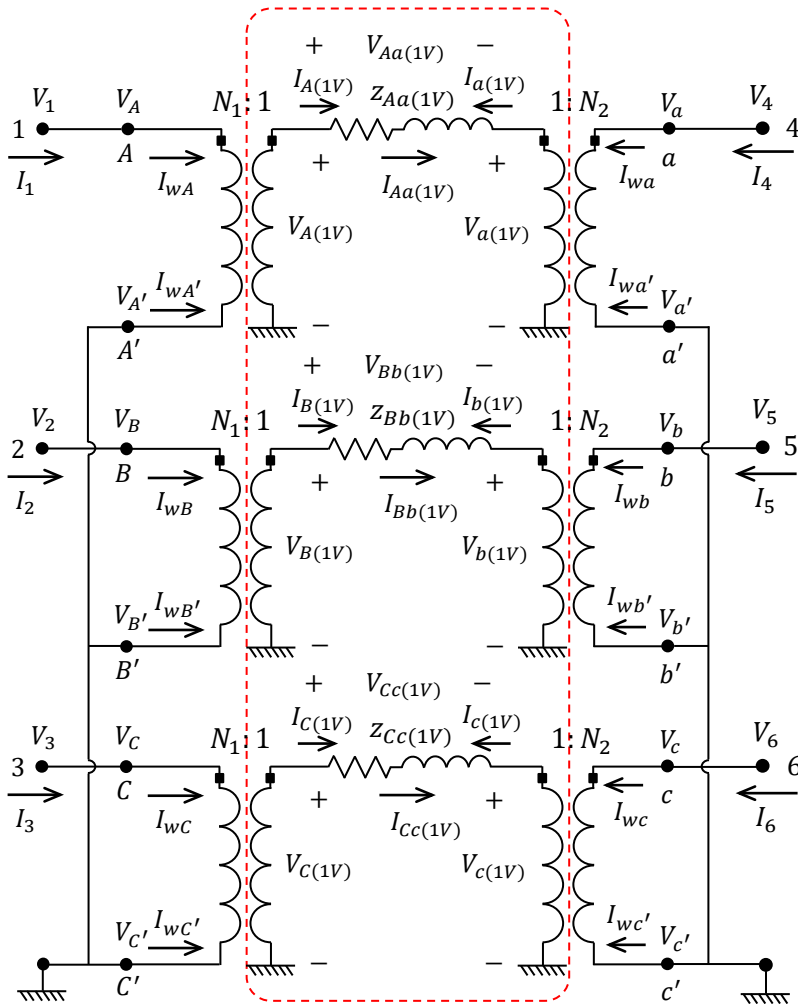
$$\Rightarrow \mathbf{V}_{sc(1V)} = \mathbf{B}^T \mathbf{V}_1, \text{ where } \mathbf{V}_{sc(1V)} = \begin{bmatrix} V_{Aa}(1V) \\ V_{Bb}(1V) \\ V_{Cc}(1V) \end{bmatrix}, \mathbf{V}_1 = \begin{bmatrix} V_A(1V) \\ V_a(1V) \\ V_B(1V) \\ V_b(1V) \\ V_C(1V) \\ V_c(1V) \end{bmatrix}$$

$$\textcircled{3} \quad \begin{cases} V_{Aa}(1V) = Z_{Aa}(1V) I_{Aa}(1V) \\ V_{Bb}(1V) = Z_{Bb}(1V) I_{Bb}(1V) \\ V_{Cc}(1V) = Z_{Cc}(1V) I_{Cc}(1V) \end{cases} \Rightarrow \begin{bmatrix} V_{Aa}(1V) \\ V_{Bb}(1V) \\ V_{Cc}(1V) \end{bmatrix} = \begin{bmatrix} Z_{Aa}(1V) & 0 & 0 \\ 0 & Z_{Bb}(1V) & 0 \\ 0 & 0 & Z_{Cc}(1V) \end{bmatrix} \begin{bmatrix} I_{Aa}(1V) \\ I_{Bb}(1V) \\ I_{Cc}(1V) \end{bmatrix}$$

$$\Rightarrow \mathbf{V}_{sc(1V)} = \mathbf{Z}_{sc(1V)} \mathbf{I}_{sc(1V)} \Rightarrow \mathbf{Y}_{sc(1V)} \mathbf{V}_{sc(1V)} = \mathbf{I}_{sc(1V)}, \text{ where } \mathbf{Y}_{sc(1V)} = (\mathbf{Z}_{sc(1V)})^{-1}$$

Modeling A Transformer in OpenDSS

7. Three-phase Grounded Wye-Grounded Wye Transformer



Let $I_1 = Y_1 V_1$, where Y_1 is the admittance matrix on a one turn or one voltage base. What is Y_1 ?

$$\left. \begin{aligned} I_1 &= Y_1 V_1 \\ I_1 &= B I_{sc(1V)} \end{aligned} \right\}$$

$$\Rightarrow \left. \begin{aligned} B I_{sc(1V)} &= Y_1 V_1 \\ V_{sc(1V)} &= B^T V_1 \end{aligned} \right\}$$

$$\Rightarrow \left. \begin{aligned} B I_{sc(1V)} &= Y_1 (B^T)^{-1} V_{sc(1V)} \\ I_{sc(1V)} &= Y_{sc(1V)} V_{sc(1V)} \end{aligned} \right\}$$

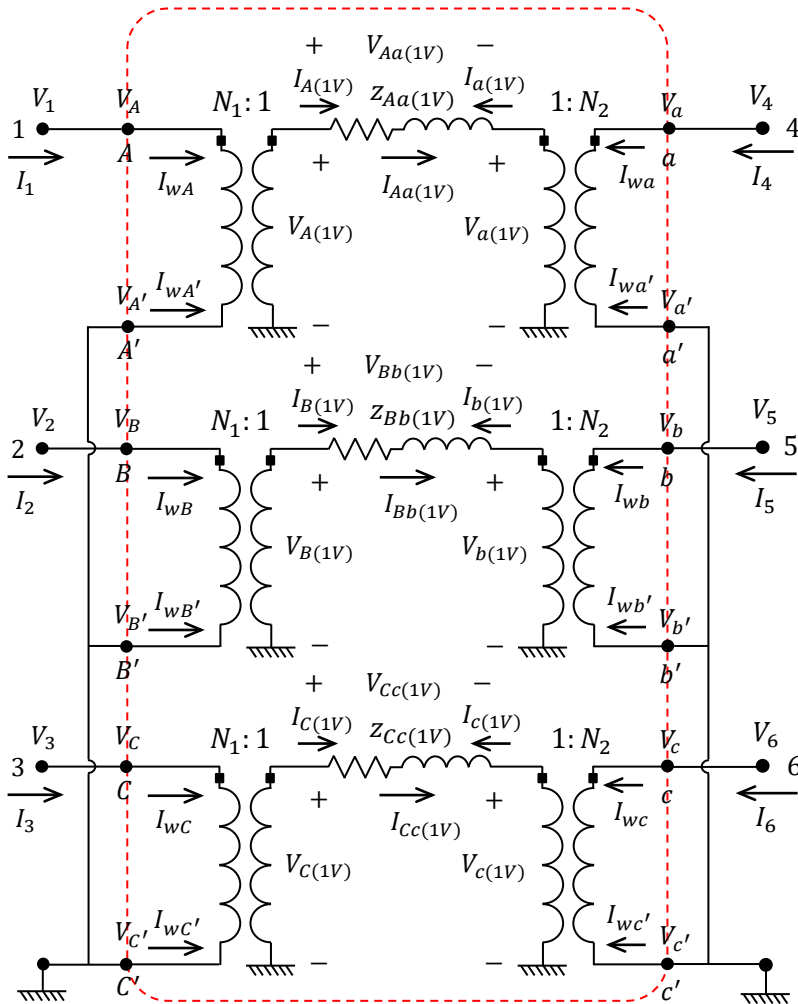
$$\Rightarrow B Y_{sc(1V)} V_{sc(1V)} = Y_1 (B^T)^{-1} V_{sc(1V)}$$

$$\Rightarrow Y_1 = B Y_{sc(1V)} B^T$$

Modeling A Transformer in OpenDSS

7. Three-phase Grounded Wye-Grounded Wye Transformer

Step 2: Consider winding turns ratio.

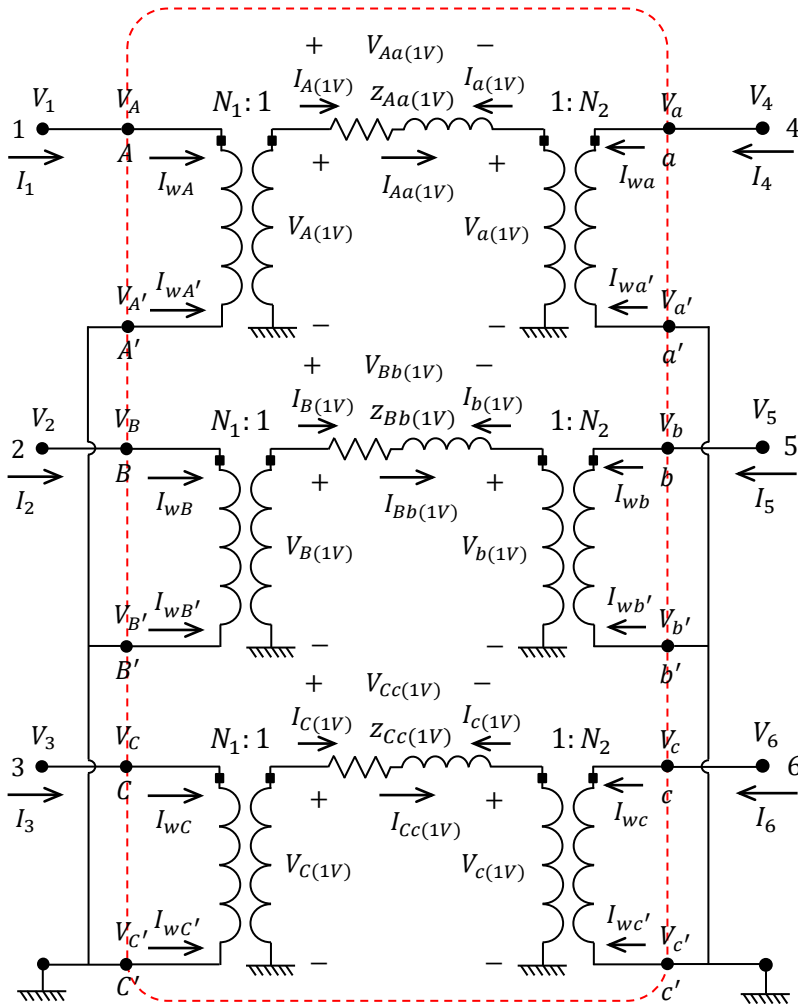


$$\begin{aligned}
 & \text{①} \left\{ \begin{aligned}
 I_{wA} &= I_{A(1V)} / N_1 \\
 I_{wA'} &= -I_{A(1V)} / N_1 \\
 I_{wa} &= I_{a(1V)} / N_2 \\
 I_{wa'} &= -I_{a(1V)} / N_2 \\
 I_{wB} &= I_{B(1V)} / N_1 \\
 I_{wB'} &= -I_{B(1V)} / N_1 \\
 I_{wb} &= I_{b(1V)} / N_2 \\
 I_{wb'} &= -I_{b(1V)} / N_2 \\
 I_{wC} &= I_{C(1V)} / N_1 \\
 I_{wC'} &= -I_{C(1V)} / N_1 \\
 I_{wc} &= I_{c(1V)} / N_2 \\
 I_{wc'} &= -I_{c(1V)} / N_2
 \end{aligned} \right. \Rightarrow \begin{bmatrix} I_{wA} \\ I_{wA'} \\ I_{wa} \\ I_{wa'} \\ I_{wB} \\ I_{wB'} \\ I_{wb} \\ I_{wb'} \\ I_{wC} \\ I_{wC'} \\ I_{wc} \\ I_{wc'} \end{bmatrix} = \begin{bmatrix} \frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{N_2} & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{N_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{N_1} & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{N_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{N_2} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{N_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{N_1} & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{N_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{N_2} \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{N_2} \end{bmatrix} \begin{bmatrix} I_{A(1V)} \\ I_{a(1V)} \\ I_{B(1V)} \\ I_{b(1V)} \\ I_{C(1V)} \\ I_{c(1V)} \end{bmatrix}
 \end{aligned}$$

$$\Rightarrow I_w = N I_1, \text{ where } I_w = \begin{bmatrix} I_{wA} \\ I_{wA'} \\ I_{wa} \\ I_{wa'} \\ I_{wB} \\ I_{wB'} \\ I_{wb} \\ I_{wb'} \\ I_{wC} \\ I_{wC'} \\ I_{wc} \\ I_{wc'} \end{bmatrix}, N = \begin{bmatrix} \frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{N_2} & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{N_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{N_1} & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{N_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{N_2} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{N_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{N_1} & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{N_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{N_2} \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{N_2} \end{bmatrix}$$

Modeling A Transformer in OpenDSS

7. Three-phase Grounded Wye-Grounded Wye Transformer



②

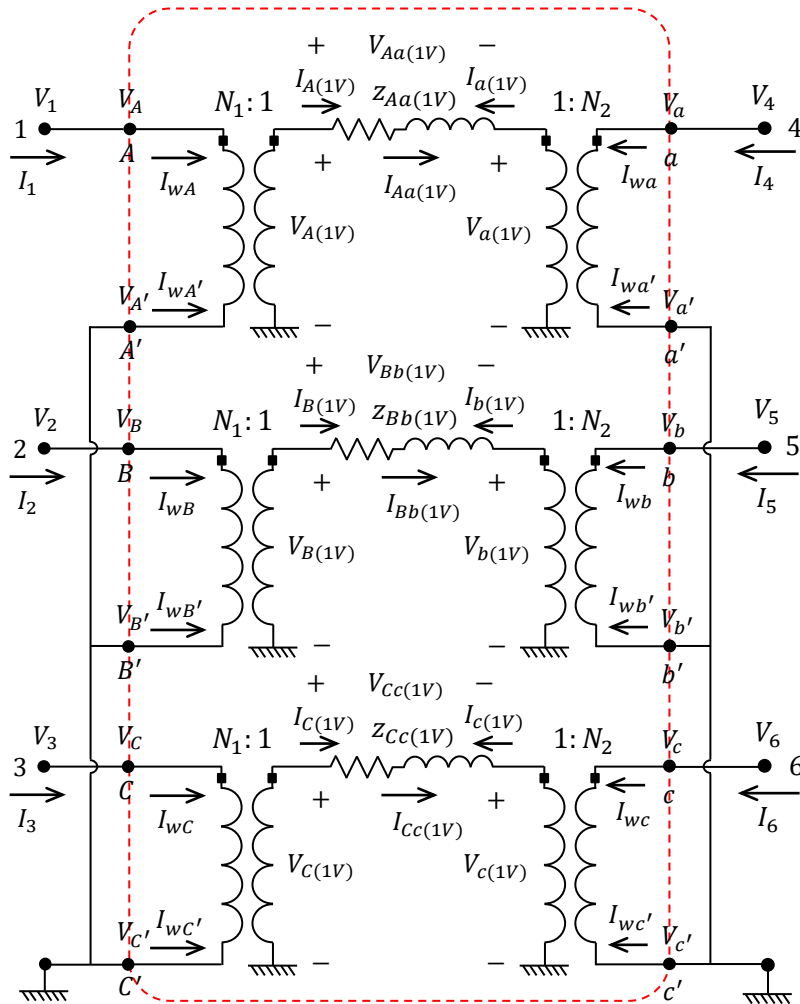
$$\begin{cases} V_{wA} - V_{wA'} = V_{A(1V)}N_1 \\ V_{wa} - V_{wa'} = V_{a(1V)}N_2 \\ V_{wB} - V_{wB'} = V_{B(1V)}N_1 \\ V_{wb} - V_{wb'} = V_{b(1V)}N_2 \\ V_{wC} - V_{wC'} = V_{C(1V)}N_1 \\ V_{wc} - V_{wc'} = V_{c(1V)}N_2 \end{cases}$$

$$\begin{bmatrix} V_{A(1V)} \\ V_{a(1V)} \\ V_{B(1V)} \\ V_{b(1V)} \\ V_{C(1V)} \\ V_{c(1V)} \end{bmatrix} = \begin{bmatrix} \frac{1}{N_1} & -\frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{N_2} & -\frac{1}{N_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{N_1} & -\frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{N_2} & -\frac{1}{N_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{N_1} & -\frac{1}{N_1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{N_2} & -\frac{1}{N_2} \end{bmatrix} \begin{bmatrix} V_{wA} \\ V_{wA'} \\ V_{wa} \\ V_{wa'} \\ V_{wB} \\ V_{wB'} \\ V_{wb} \\ V_{wb'} \\ V_{wC} \\ V_{wC'} \\ V_{wc} \\ V_{wc'} \end{bmatrix}$$

$$\Rightarrow V_1 = N^T V_w, \text{ where } V_w = \begin{bmatrix} V_{wA} \\ V_{wA'} \\ V_{wa} \\ V_{wa'} \\ V_{wB} \\ V_{wB'} \\ V_{wb} \\ V_{wb'} \\ V_{wC} \\ V_{wC'} \\ V_{wc} \\ V_{wc'} \end{bmatrix}$$

Modeling A Transformer in OpenDSS

7. Three-phase Grounded Wye-Grounded Wye Transformer



Next, let $I_w = Y_w V_w$. How to obtain Y_w ?

$$\left. \begin{aligned} I_w &= Y_w V_w \\ I_w &= N I_1 \\ V_1 &= N^T V_w \end{aligned} \right\}$$

$$\Rightarrow \left. \begin{aligned} N I_1 &= Y_w (N^T)^{-1} V_1 \\ I_1 &= Y_1 V_1 \end{aligned} \right\}$$

$$\Rightarrow N Y_1 V_1 = Y_w (N^T)^{-1} V_1$$

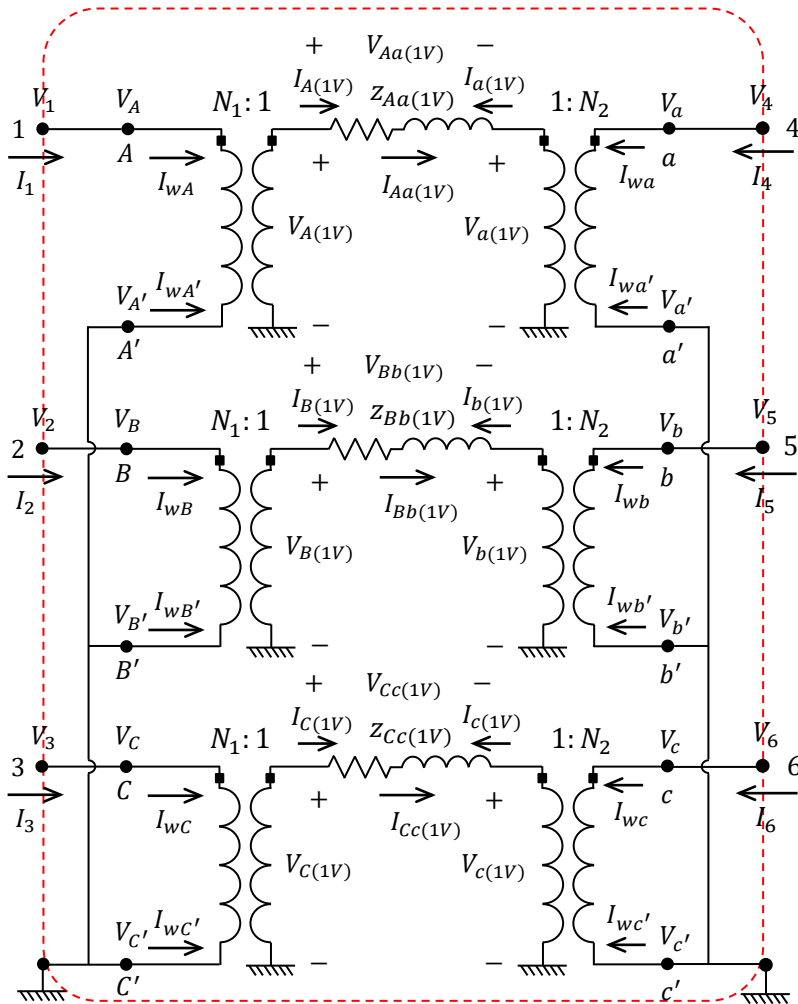
$$\Rightarrow \left. \begin{aligned} Y_w &= N Y_1 N^T \\ Y_1 &= B Y_{sc(1V)} B^T \end{aligned} \right\}$$

$$\Rightarrow Y_w = N B Y_{sc(1V)} B^T N^T$$

Modeling A Transformer in OpenDSS

7. Three-phase Grounded Wye-Grounded Wye Transformer

Step 3: Consider winding connections



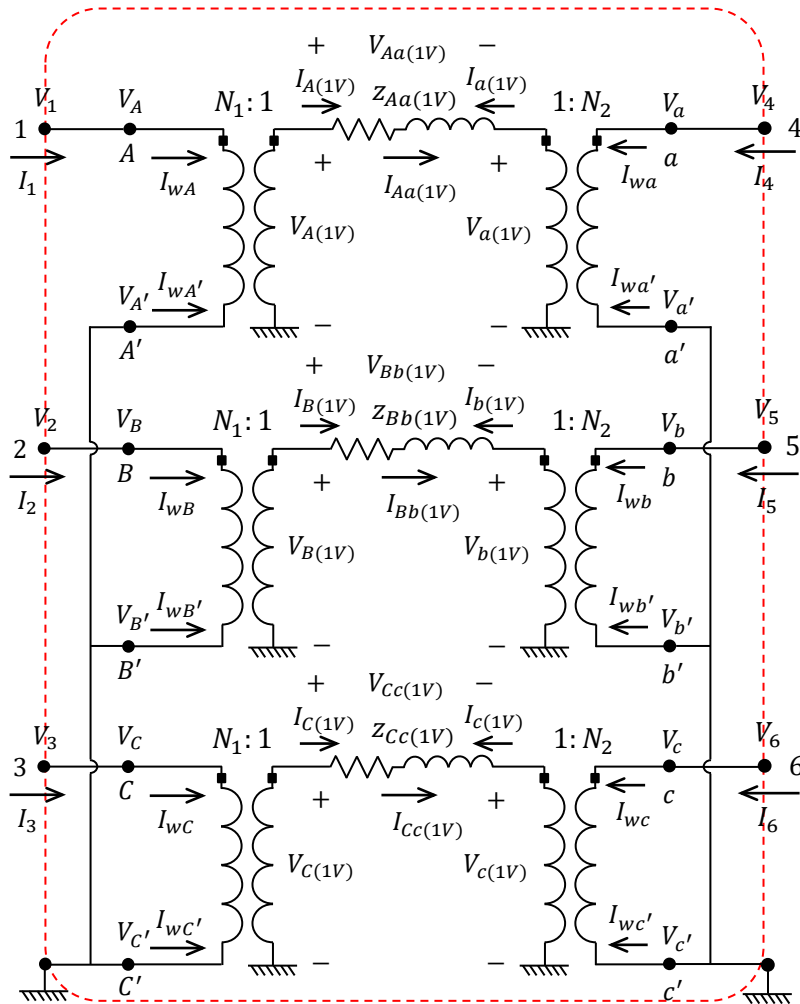
$$\textcircled{1} \begin{cases} I_1 = I_{wA} \\ I_2 = I_{wB} \\ I_3 = I_{wC} \\ I_4 = I_{wa} \\ I_5 = I_{wb} \\ I_6 = I_{wc} \end{cases}$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_{wA} \\ I_{wA'} \\ I_{wa} \\ I_{wa'} \\ I_{wB} \\ I_{wB'} \\ I_{wb} \\ I_{wb'} \\ I_{wC} \\ I_{wC'} \\ I_{wc} \\ I_{wc'} \end{bmatrix}$$

$$\Rightarrow I_{prim} = AI_w, \text{ where, } I_{prim} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix}, A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Modeling A Transformer in OpenDSS

7. Three-phase Grounded Wye-Grounded Wye Transformer

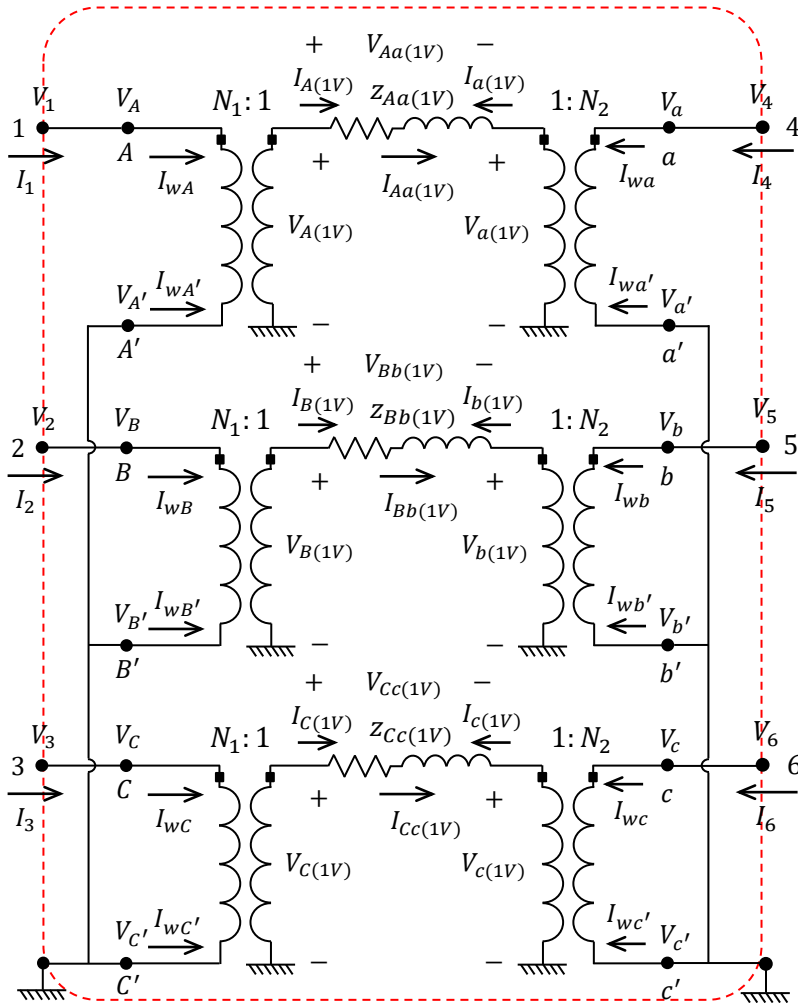


$$\textcircled{2} \left\{ \begin{array}{l} V_A = V_1 \\ V_{A'} = 0 \\ V_a = V_4 \\ V_{a'} = 0 \\ V_B = V_2 \\ V_{B'} = 0 \\ V_b = V_5 \\ V_{b'} = 0 \\ V_C = V_3 \\ V_{C'} = 0 \\ V_c = V_6 \\ V_{c'} = 0 \end{array} \right. \Rightarrow \begin{bmatrix} V_A \\ V_{A'} \\ V_a \\ V_{a'} \\ V_B \\ V_{B'} \\ V_b \\ V_{b'} \\ V_C \\ V_{C'} \\ V_c \\ V_{c'} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix}$$

$$\Rightarrow \mathbf{V}_w = \mathbf{A}^T \mathbf{V}_{prim}, \text{ where } \mathbf{V}_{prim} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix}$$

Modeling A Transformer in OpenDSS

7. Three-phase Grounded Wye-Grounded Wye Transformer



Let $I_{prim} = Y_{prim} V_{prim}$. How to obtain Y_{prim} ?

$$\left. \begin{aligned} I_{prim} &= Y_{prim} V_{prim} \\ I_{prim} &= A I_w \end{aligned} \right\}$$

$$\Rightarrow \left. \begin{aligned} A I_w &= Y_{prim} V_{prim} \\ V_w &= A^T V_{prim} \end{aligned} \right\}$$

$$\Rightarrow \left. \begin{aligned} A I_w &= Y_{prim} (A^T)^{-1} V_w \\ I_w &= Y_w V_w \end{aligned} \right\}$$

$$\Rightarrow \left. \begin{aligned} A Y_w V_w &= Y_{prim} (A^T)^{-1} V_w \\ Y_w &= N B Y_{sc(1V)} B^T N^T \end{aligned} \right\}$$

$$\Rightarrow Y_{prim} = A N B Y_{sc(1V)} B^T N^T A^T$$

Modeling A Transformer in OpenDSS

7. Three-phase Grounded Wye-Grounded Wye Transformer

An example:

// Define a three-phase transformer.

New Transformer.xfmr phases=3 windings=2 xhl=5

~ wdg=1 bus=K.1.2.3.0 conn=wye kV=115 kva=500 %r=1

~ wdg=2 bus=M.1.2.3.0 conn=wye kV=4.16 kva=500 %r=1

Specify which winding
will be edited

Transformer name

Number of
phases

Number of
windings

Percent reactance
high-to-low

Winding
connection

Rated voltage

Base kVA
rating

Percent
resistance

Computing Y_{prim} using Matlab:

$$Y_{Aa(1V)} = Y_{Bb(1V)} = Y_{Cc(1V)} = \frac{1}{[r(pu) + jx(pu)] * \frac{1^2}{S_t/3}} = \frac{500000}{3 * (0.02 + j0.05)} = 1149425.2873 - j2873563.2184$$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix}$$

$$N^T = \begin{bmatrix} \frac{1}{115000/\sqrt{3}} & -\frac{1}{115000/\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4160/\sqrt{3}} & -\frac{1}{4160/\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{115000/\sqrt{3}} & -\frac{1}{115000/\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4160/\sqrt{3}} & -\frac{1}{4160/\sqrt{3}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{115000/\sqrt{3}} & -\frac{1}{115000/\sqrt{3}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4160/\sqrt{3}} & -\frac{1}{4160/\sqrt{3}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4160/\sqrt{3}} & -\frac{1}{4160/\sqrt{3}} \end{bmatrix}$$

Modeling A Transformer in OpenDSS

7. Three-phase Grounded Wye-Grounded Wye Transformer

$$\mathbf{Y}_{sc(1V)} = \begin{bmatrix} y_{Aa(1V)} & 0 & 0 \\ 0 & y_{Bb(1V)} & 0 \\ 0 & 0 & y_{Cc(1V)} \end{bmatrix} = \begin{bmatrix} 1149425.2873 - j2873563.2184 & 0 & 0 \\ 0 & 1149425.2873 - j2873563.2184 & 0 \\ 0 & 0 & 1149425.2873 - j2873563.2184 \end{bmatrix}$$

Calculate $\mathbf{Y}_{prim} = \mathbf{ANBY}_{sc(1V)}\mathbf{B}^T\mathbf{N}^T\mathbf{A}^T =$

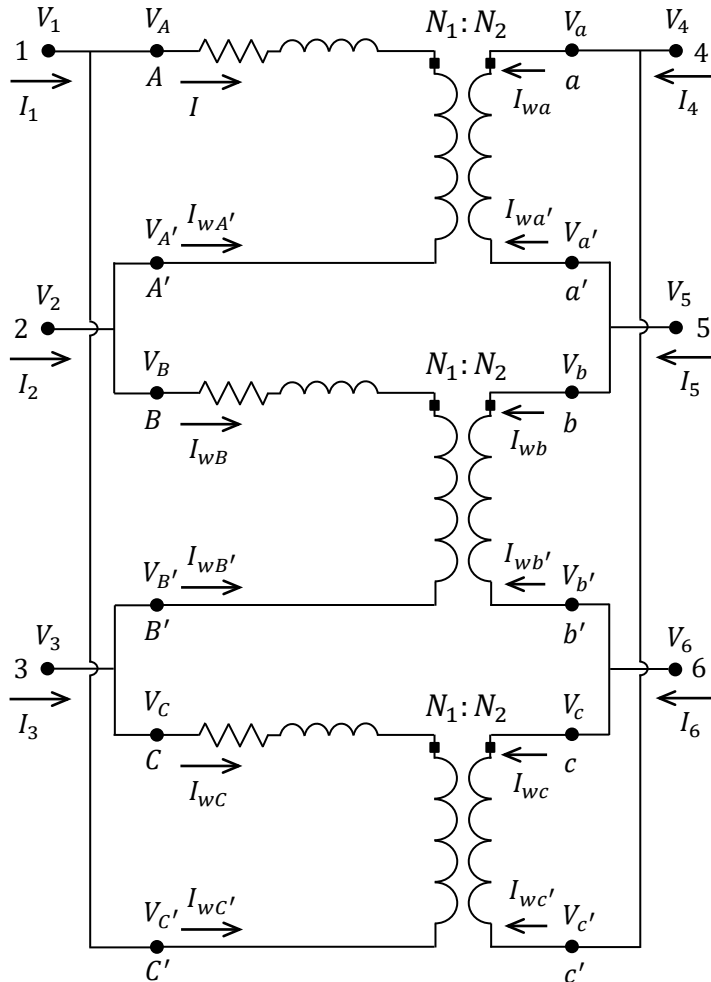
$$\begin{bmatrix} 0.000260739 - j0.00065184 & 0 + j0 & 0 + j0 & -0.00720793 + j0.01801984 & 0 + j0 & 0 + j0 \\ 0 + j0 & 0.000260739 - j0.00065184 & 0 + j0 & 0 + j0 & -0.00720793 + j0.01801984 & 0 + j0 \\ 0 + j0 & 0 + j0 & 0.000260739 - j0.00065184 & 0 + j0 & 0 + j0 & -0.00720793 + j0.01801984 \\ -0.00720793 + j0.01801984 & 0 + j0 & 0 + j0 & 0.19925780 - j0.49814451 & 0 + j0 & 0 + j0 \\ 0 + j0 & -0.00720793 + j0.01801984 & 0 + j0 & 0.19925780 - j0.49814451 & 0 + j0 & 0 + j0 \\ 0 + j0 & 0 + j0 & -0.00720793 + j0.01801984 & 0 + j0 & 0.19925780 - j0.49814451 & 0.19925780 - j0.49814451 \end{bmatrix}$$

Exported $\mathbf{Y}_{prim}$ from OpenDSS:

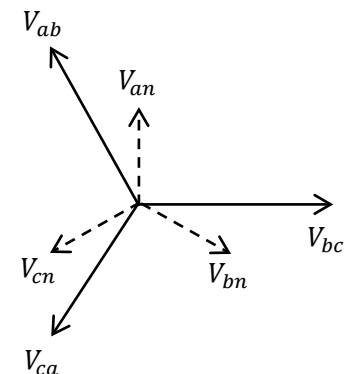
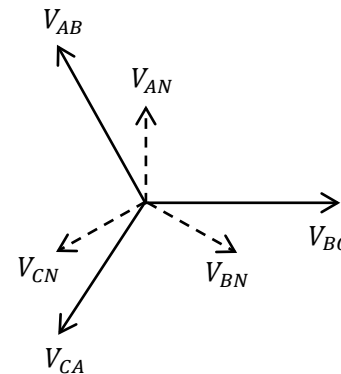
	G			B								
	↓			↓								
1	0.000261	-0.00065	0	0	0	0	-0.00721	0.01802	0	0	0	0
2	0	0	0.000261	-0.00065	0	0	0	0	-0.00721	0.01802	0	0
3	0	0	0	0	0.000261	-0.00065	0	0	0	0	-0.00721	0.01802
4	-0.00721	0.01802	0	0	0	0	0.199258	-0.49814	0	0	0	0
5	0	0	-0.00721	0.01802	0	0	0	0	0.199258	-0.49814	0	0
6	0	0	0	0	-0.00721	0.01802	0	0	0	0	0.199258	-0.49814
	1	2	3		4	5		6				

Modeling A Transformer in OpenDSS

8. Three-phase Delta-Delta Transformer



- The HV side always comes before the LV side, regardless of which is the primary winding.
- HV winding is taken as a reference.
- Phase rotation is always counterclockwise.
- 1 = 30°, 2 = 60°, 3 = 90°, 6 = 180° and 12 = 0° or 360°.

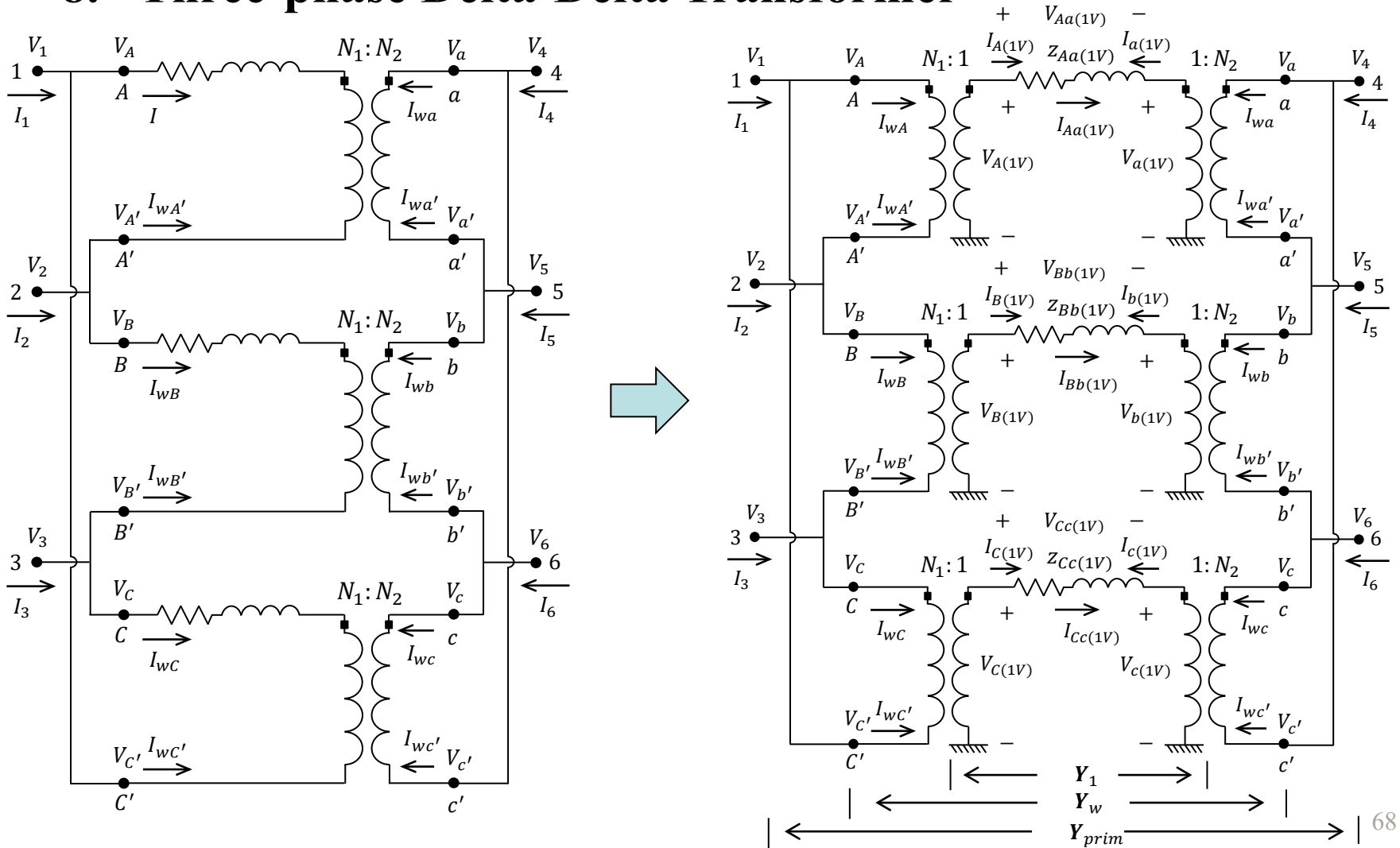


Dd0

67

Modeling A Transformer in OpenDSS

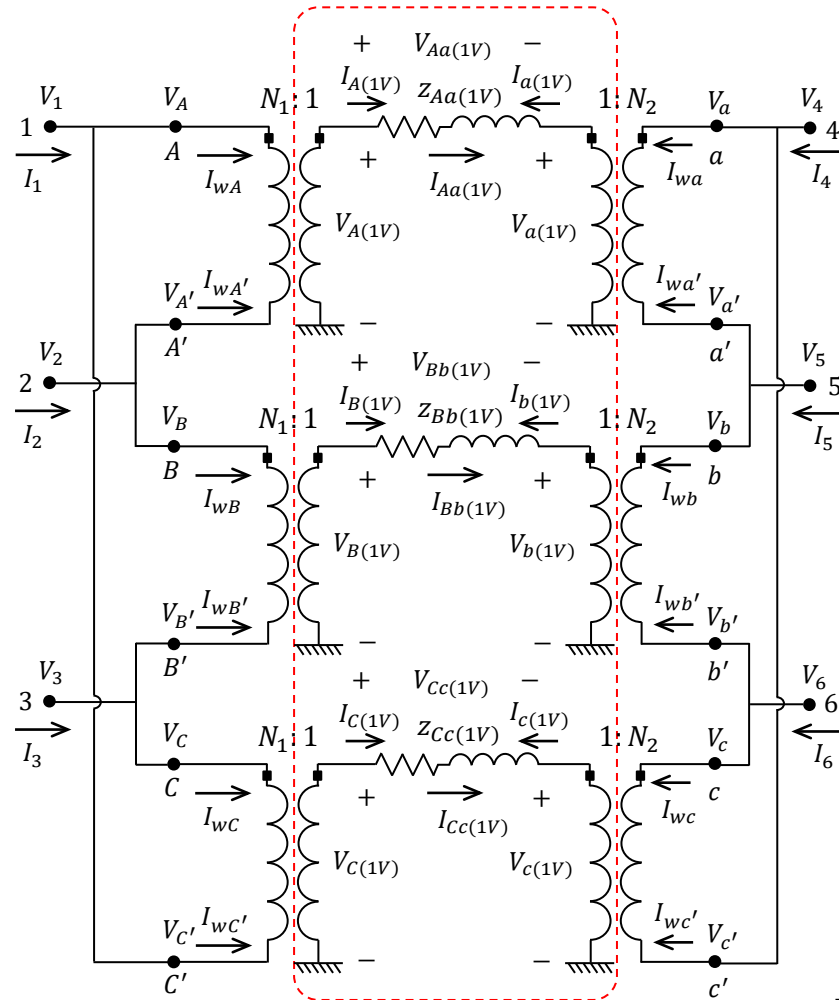
8. Three-phase Delta-Delta Transformer



Modeling A Transformer in OpenDSS

8. Three-phase Delta-Delta Transformer

Step 1: Compute the nodal admittance matrix on a one turn or one voltage base.



$$\textcircled{1} \quad \begin{cases} I_A(1V) = I_{Aa}(1V) \\ I_a(1V) = -I_{Aa}(1V) \\ I_B(1V) = I_{Bb}(1V) \\ I_b(1V) = -I_{Bb}(1V) \\ I_C(1V) = I_{Cc}(1V) \\ I_c(1V) = -I_{Cc}(1V) \end{cases} \Rightarrow \begin{bmatrix} I_A(1V) \\ I_a(1V) \\ I_B(1V) \\ I_b(1V) \\ I_C(1V) \\ I_c(1V) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} I_{Aa}(1V) \\ I_{Bb}(1V) \\ I_{Cc}(1V) \end{bmatrix}$$

$$\Rightarrow I_1 = B I_{sc(1V)}, \text{ where } I_1 = \begin{bmatrix} I_A(1V) \\ I_a(1V) \\ I_B(1V) \\ I_b(1V) \\ I_C(1V) \\ I_c(1V) \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix}, I_{sc(1V)} = \begin{bmatrix} I_{Aa}(1V) \\ I_{Bb}(1V) \\ I_{Cc}(1V) \end{bmatrix}$$

$$\textcircled{2} \quad \begin{cases} V_{Aa}(1V) = V_A(1V) - V_a(1V) \\ V_{Bb}(1V) = V_B(1V) - V_b(1V) \\ V_{Cc}(1V) = V_C(1V) - V_c(1V) \end{cases} \Rightarrow \begin{bmatrix} V_{Aa}(1V) \\ V_{Bb}(1V) \\ V_{Cc}(1V) \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} V_A(1V) \\ V_a(1V) \\ V_B(1V) \\ V_b(1V) \\ V_C(1V) \\ V_c(1V) \end{bmatrix}$$

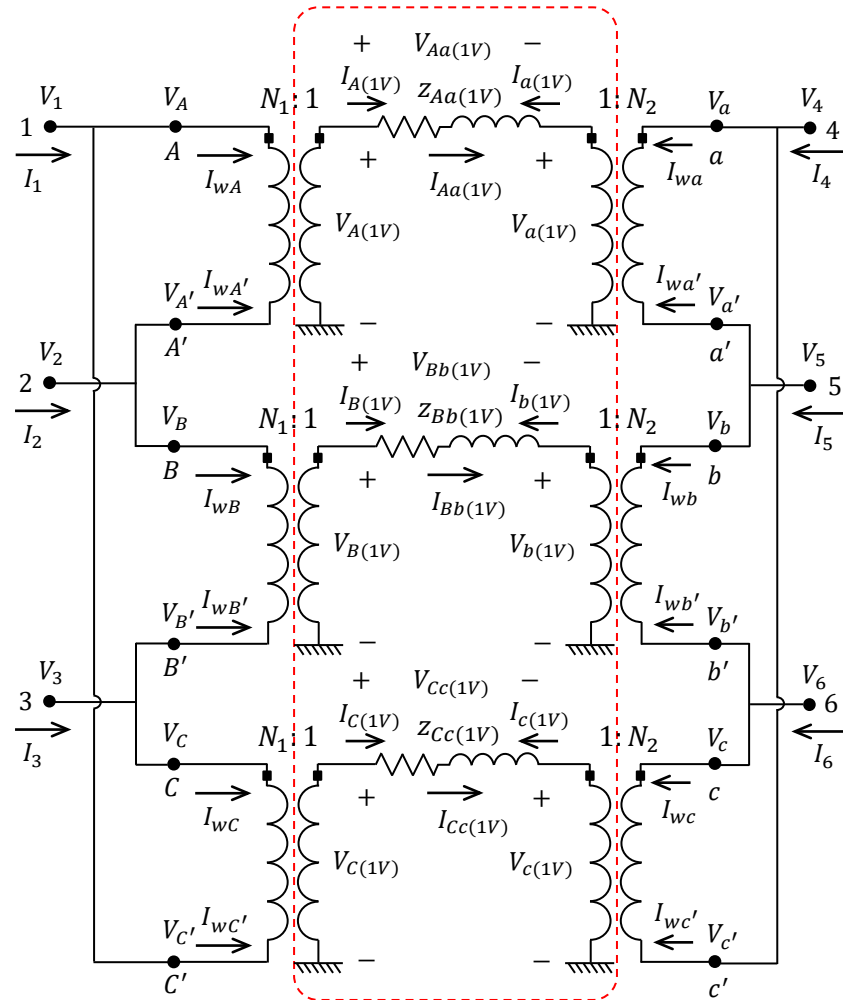
$$\Rightarrow V_{sc(1V)} = B^T V_1, \text{ where } V_{sc(1V)} = \begin{bmatrix} V_{Aa}(1V) \\ V_{Bb}(1V) \\ V_{Cc}(1V) \end{bmatrix}, V_1 = \begin{bmatrix} V_A(1V) \\ V_a(1V) \\ V_B(1V) \\ V_b(1V) \\ V_C(1V) \\ \dot{V}_c(1V) \end{bmatrix}$$

$$\textcircled{3} \quad \begin{cases} V_{Aa}(1V) = Z_{Aa}(1V) I_{Aa}(1V) \\ V_{Bb}(1V) = Z_{Bb}(1V) I_{Bb}(1V) \\ V_{Cc}(1V) = Z_{Cc}(1V) I_{Cc}(1V) \end{cases} \Rightarrow \begin{bmatrix} V_{Aa}(1V) \\ V_{Bb}(1V) \\ V_{Cc}(1V) \end{bmatrix} = \begin{bmatrix} Z_{Aa}(1V) & 0 & 0 \\ 0 & Z_{Bb}(1V) & 0 \\ 0 & 0 & Z_{Cc}(1V) \end{bmatrix} \begin{bmatrix} I_{Aa}(1V) \\ I_{Bb}(1V) \\ I_{Cc}(1V) \end{bmatrix}$$

$$\Rightarrow V_{sc(1V)} = Z_{sc(1V)} I_{sc(1V)} \Rightarrow Y_{sc(1V)} V_{sc(1V)} = I_{sc(1V)}, \text{ where } Y_{sc(1V)} = (Z_{sc(1V)})^{-1}$$

Modeling A Transformer in OpenDSS

8. Three-phase Delta-Delta Transformer



Let $I_1 = Y_1 V_1$, where Y_1 is the admittance matrix on a one turn or one voltage base. What is Y_1 ?

$$\left. \begin{aligned} I_1 &= Y_1 V_1 \\ I_1 &= B I_{sc(1V)} \end{aligned} \right\}$$

$$\Rightarrow \left. \begin{aligned} B I_{sc(1V)} &= Y_1 V_1 \\ V_{sc(1V)} &= B^T V_1 \end{aligned} \right\}$$

$$\Rightarrow \left. \begin{aligned} B I_{sc(1V)} &= Y_1 (B^T)^{-1} V_{sc(1V)} \\ I_{sc(1V)} &= Y_{sc(1V)} V_{sc(1V)} \end{aligned} \right\}$$

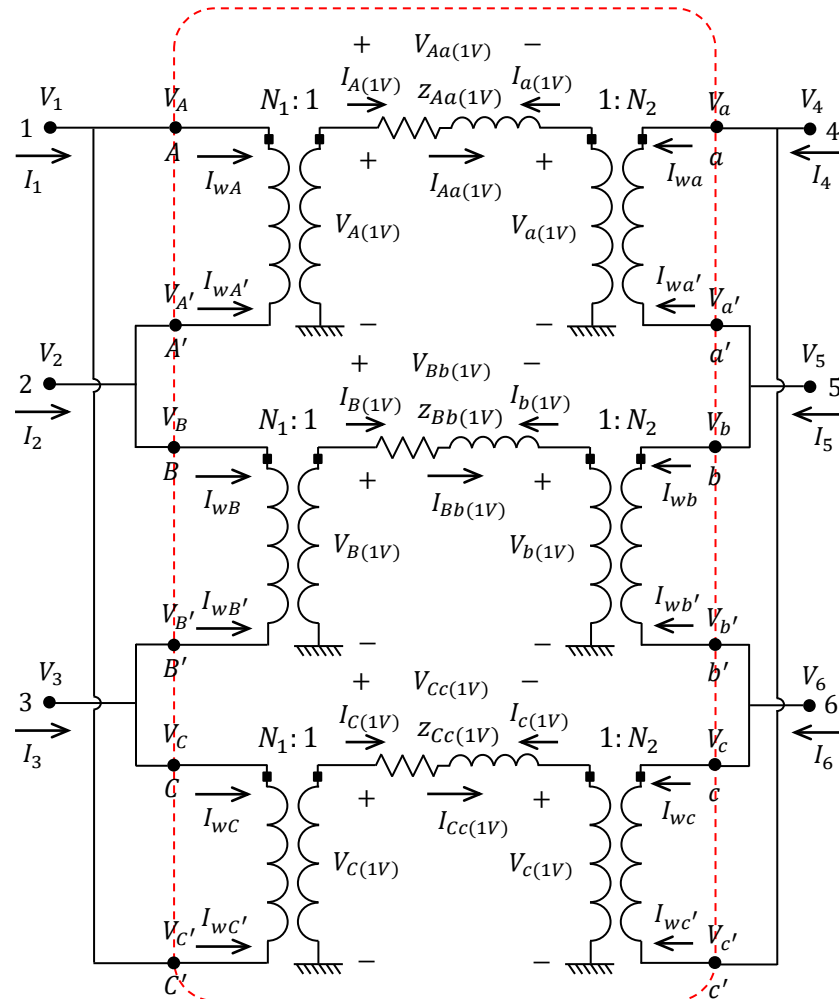
$$\Rightarrow B Y_{sc(1V)} V_{sc(1V)} = Y_1 (B^T)^{-1} V_{sc(1V)}$$

$$\Rightarrow Y_1 = B Y_{sc(1V)} B^T$$

Modeling A Transformer in OpenDSS

8. Three-phase Delta-Delta Transformer

Step 2: Consider winding turns ratio.



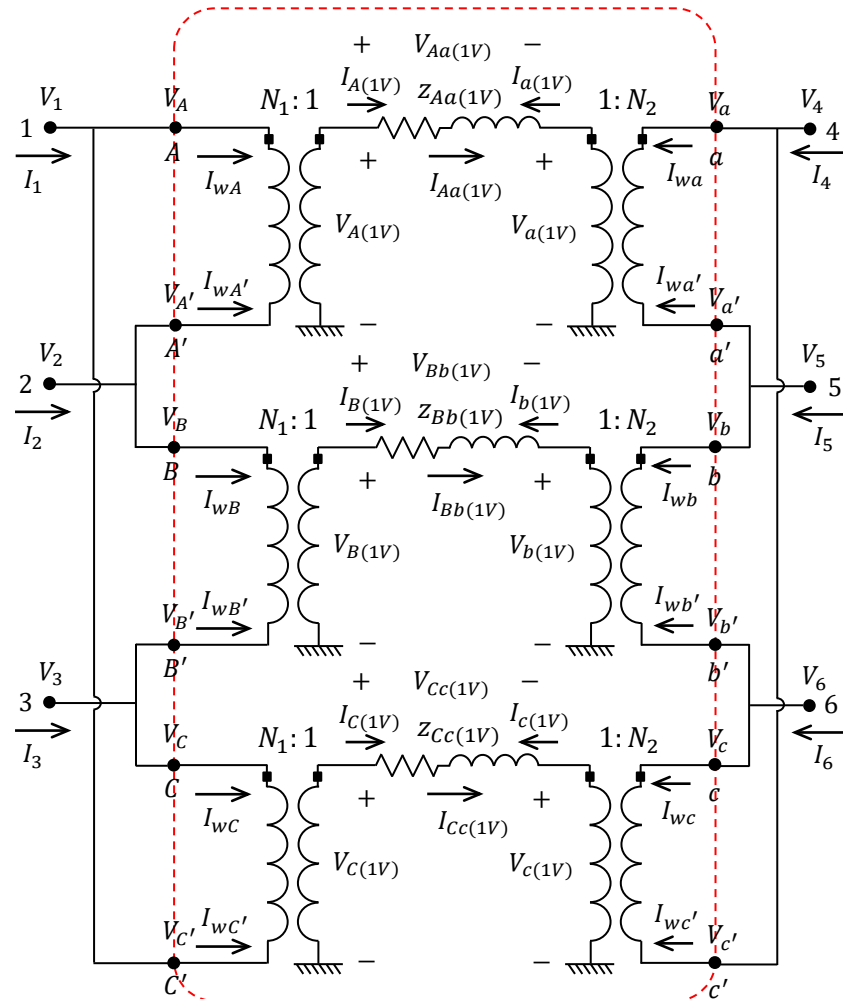
①

$$\begin{cases} I_{wA} = I_{A(1V)}/N_1 \\ I_{wA'} = -I_{A(1V)}/N_1 \\ I_{wB} = I_{B(1V)}/N_1 \\ I_{wB'} = -I_{B(1V)}/N_1 \\ I_{wC} = I_{C(1V)}/N_1 \\ I_{wC'} = -I_{C(1V)}/N_1 \end{cases} \Rightarrow \begin{bmatrix} I_{wA} \\ I_{wA'} \\ I_{wB} \\ I_{wB'} \\ I_{wC} \\ I_{wC'} \end{bmatrix} = \begin{bmatrix} \frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{N_2} & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{N_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{N_1} & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{N_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{N_2} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{N_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{N_1} & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{N_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{N_2} \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{N_2} \end{bmatrix} \begin{bmatrix} I_{A(1V)} \\ I_{a(1V)} \\ I_{B(1V)} \\ I_{b(1V)} \\ I_{C(1V)} \\ I_{c(1V)} \end{bmatrix}$$

$$\Rightarrow I_w = NI_1, \text{ where } I_w = \begin{bmatrix} I_{wA} \\ I_{wA'} \\ I_{wB} \\ I_{wB'} \\ I_{wC} \\ I_{wC'} \end{bmatrix}, N = \begin{bmatrix} \frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{N_2} & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{N_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{N_1} & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{N_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{N_2} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{N_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{N_1} & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{N_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{N_2} \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{N_2} \end{bmatrix}$$

Modeling A Transformer in OpenDSS

8. Three-phase Delta-Delta Transformer



2

$$\begin{cases} V_{wA} - V_{wA'} = V_{A(1V)}N_1 \\ V_{wa} - V_{wa'} = V_{a(1V)}N_2 \\ V_{wB} - V_{wB'} = V_{B(1V)}N_1 \\ V_{wb} - V_{wb'} = V_{b(1V)}N_2 \\ V_{wC} - V_{wC'} = V_{C(1V)}N_1 \\ V_{wc} - V_{wc'} = V_{c(1V)}N_2 \end{cases}$$



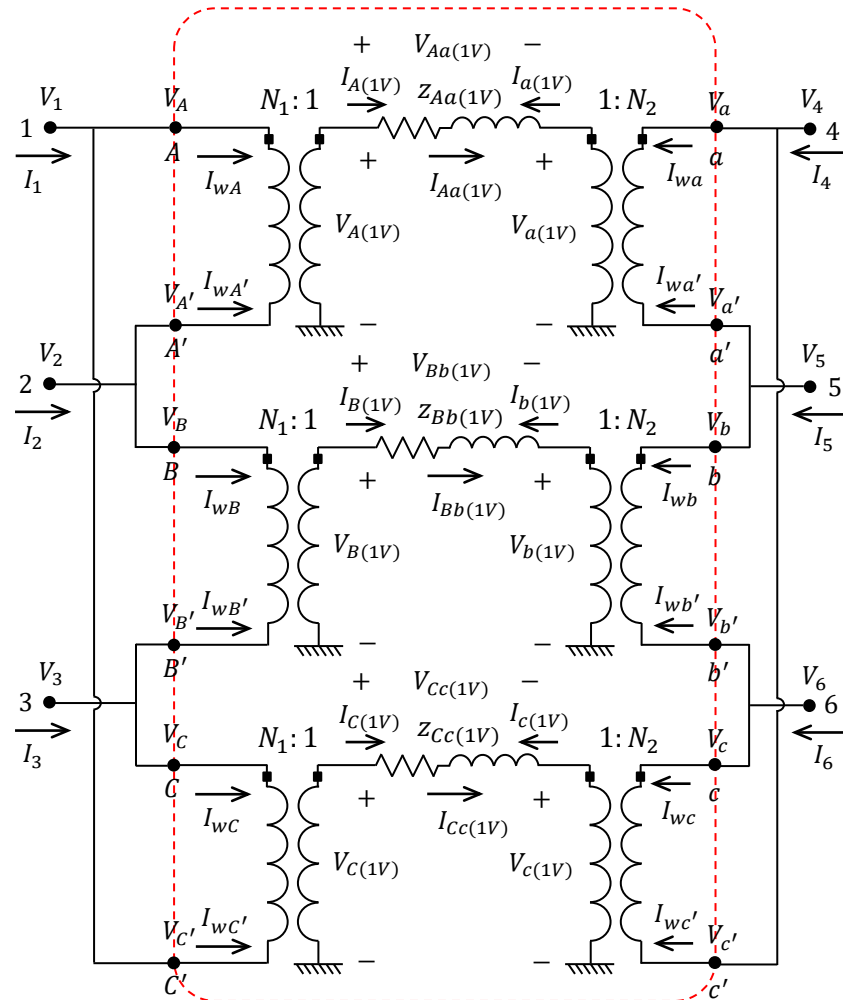
$$\begin{bmatrix} V_{A(1V)} \\ V_{a(1V)} \\ V_{B(1V)} \\ V_{b(1V)} \\ V_{C(1V)} \\ V_{c(1V)} \end{bmatrix} = \begin{bmatrix} \frac{1}{N_1} & -\frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{N_2} & -\frac{1}{N_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{N_1} & -\frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{N_2} & -\frac{1}{N_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{N_1} & -\frac{1}{N_1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{N_2} & -\frac{1}{N_2} \end{bmatrix} \begin{bmatrix} V_{wA} \\ V_{wA'} \\ V_{wa} \\ V_{wa'} \\ V_{wB} \\ V_{wB'} \\ V_{wb} \\ V_{wb'} \\ V_{wC} \\ V_{wC'} \\ V_{wc} \\ V_{wc'} \end{bmatrix}$$



$$V_1 = N^T V_w, \text{ where } V_w = \begin{bmatrix} V_{wA} \\ V_{wA'} \\ V_{wa} \\ V_{wa'} \\ V_{wB} \\ V_{wB'} \\ V_{wb} \\ V_{wb'} \\ V_{wC} \\ V_{wC'} \\ V_{wc} \\ V_{wc'} \end{bmatrix}$$

Modeling A Transformer in OpenDSS

8. Three-phase Delta-Delta Transformer



Next, let $I_w = Y_w V_w$. How to obtain Y_w ?

$$\left. \begin{aligned} I_w &= Y_w V_w \\ I_w &= N I_1 \\ V_1 &= N^T V_w \end{aligned} \right\}$$

$$\Rightarrow \left. \begin{aligned} N I_1 &= Y_w (N^T)^{-1} V_1 \\ I_1 &= Y_1 V_1 \end{aligned} \right\}$$

$$\Rightarrow N Y_1 V_1 = Y_w (N^T)^{-1} V_1$$

$$\Rightarrow \left. \begin{aligned} Y_w &= N Y_1 N^T \\ Y_1 &= B Y_{sc(1V)} B^T \end{aligned} \right\}$$

$$\Rightarrow Y_w = N B Y_{sc(1V)} B^T N^T$$

Modeling A Transformer in OpenDSS

8. Three-phase Delta-Delta Transformer

Step 3: Consider winding connections

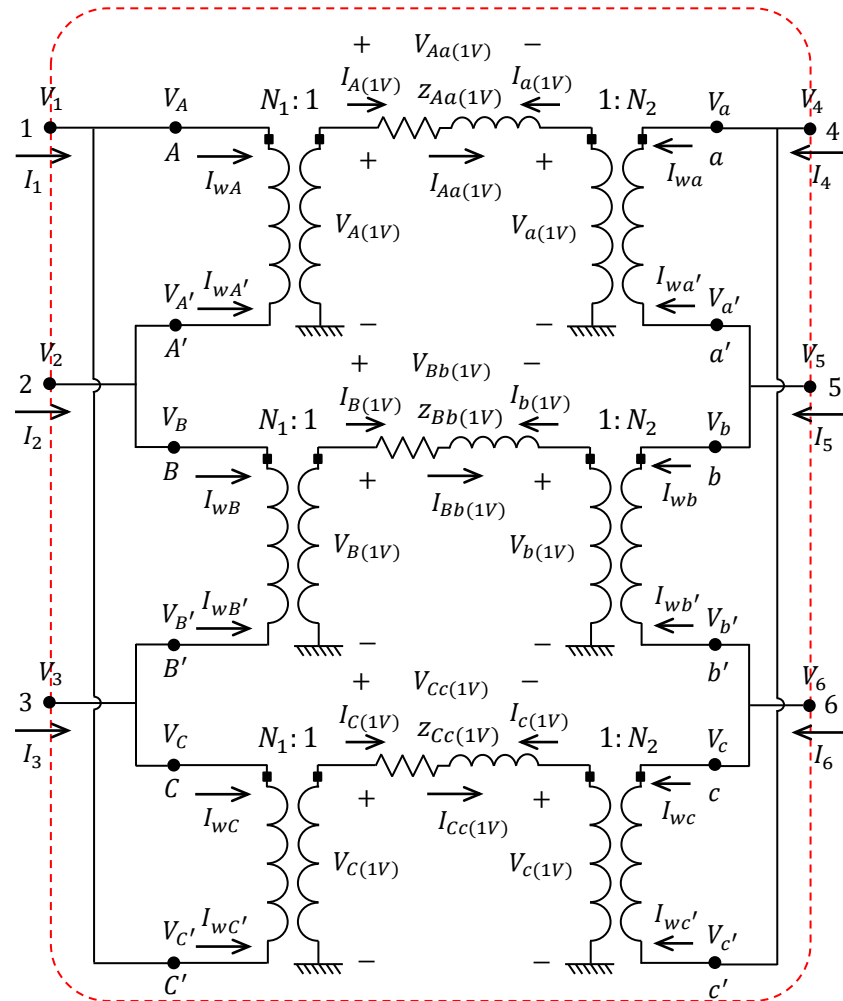
$$\textcircled{1} \begin{cases} I_1 = I_{wA} + I_{wC'} \\ I_2 = I_{wB} + I_{wA'} \\ I_3 = I_{wC} + I_{wB'} \\ I_4 = I_{wa} + I_{wc'} \\ I_5 = I_{wb} + I_{wa'} \\ I_6 = I_{wc} + I_{wb'} \end{cases}$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} I_{wA} \\ I_{wA'} \\ I_{wa} \\ I_{wa'} \\ I_{wB} \\ I_{wB'} \\ I_{wb} \\ I_{wb'} \\ I_{wC} \\ I_{wC'} \\ I_{wc} \\ I_{wc'} \end{bmatrix}$$

$$\Rightarrow I_{prim} = AI_w, \text{ where, } I_{prim} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix}, A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Modeling A Transformer in OpenDSS

8. Three-phase Delta-Delta Transformer

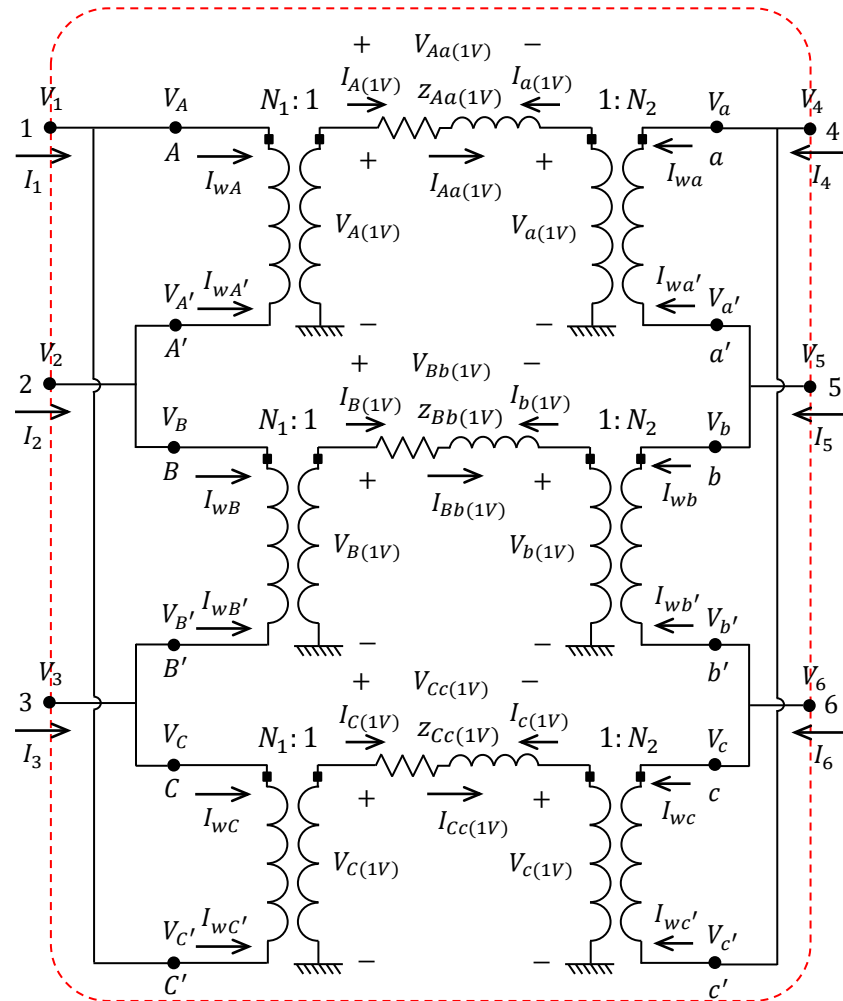


$$\textcircled{2} \left\{ \begin{array}{l} V_A = V_1 \\ V_{A'} = V_2 \\ V_a = V_4 \\ V_{a'} = V_5 \\ V_B = V_2 \\ V_{B'} = V_3 \\ V_b = V_5 \\ V_{b'} = V_6 \\ V_C = V_3 \\ V_{C'} = V_1 \\ V_c = V_6 \\ V_{c'} = V_4 \end{array} \right. \Rightarrow \begin{bmatrix} V_A \\ V_{A'} \\ V_a \\ V_{a'} \\ V_B \\ V_{B'} \\ V_b \\ V_{b'} \\ V_C \\ V_{C'} \\ V_c \\ V_{c'} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix}$$

$$\Rightarrow \mathbf{V}_w = \mathbf{A}^T \mathbf{V}_{prim}, \text{ where } \mathbf{V}_{prim} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix}$$

Modeling A Transformer in OpenDSS

8. Three-phase Delta-Delta Transformer



Let $I_{prim} = Y_{prim} V_{prim}$. How to obtain Y_{prim} ?

$$\left. \begin{aligned} I_{prim} &= Y_{prim} V_{prim} \\ I_{prim} &= A I_w \end{aligned} \right\}$$

$$\Rightarrow \left. \begin{aligned} A I_w &= Y_{prim} V_{prim} \\ V_w &= A^T V_{prim} \end{aligned} \right\}$$

$$\Rightarrow \left. \begin{aligned} A I_w &= Y_{prim} (A^T)^{-1} V_w \\ I_w &= Y_w V_w \end{aligned} \right\}$$

$$\Rightarrow \left. \begin{aligned} A Y_w V_w &= Y_{prim} (A^T)^{-1} V_w \\ Y_w &= N B Y_{sc(1V)} B^T N^T \end{aligned} \right\}$$

$$\Rightarrow Y_{prim} = A N B Y_{sc(1V)} B^T N^T A^T$$

Modeling A Transformer in OpenDSS

8. Three-phase Delta-Delta Transformer

An example:

// Define a three-phase transformer.

New Transformer.xfmr phases=3 windings=2 xhl=5

~ wdg=1 bus=K.1.2.3 conn=delta kV=115 kva=500 %r=1

~ wdg=2 bus=M.1.2.3 conn=delta kV=4.16 kva=500 %r=1

Specify which winding
will be edited

Transformer name

Number of
phases

Number of
windings

Percent reactance
high-to-low

Winding
connection

Rated voltage

Base kVA
rating

Percent
resistance

Computing Y_{prim} using Matlab:

$$Y_{Aa(1V)} = Y_{Bb(1V)} = Y_{Cc(1V)} = \frac{1}{[r(pu) + jx(pu)] * \frac{1^2}{S_t/3}} = \frac{500000}{3 * (0.02 + j0.05)} = 1149425.2873 - j2873563.2184$$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix}$$

$$N^T = \begin{bmatrix} \frac{1}{115000} & -\frac{1}{115000} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4160} & -\frac{1}{4160} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{115000} & -\frac{1}{115000} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4160} & -\frac{1}{4160} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{115000} & -\frac{1}{115000} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4160} & -\frac{1}{4160} \end{bmatrix}$$

Modeling A Transformer in OpenDSS

8. Three-phase Delta-Delta Transformer

$$\mathbf{Y}_{sc(1V)} = \begin{bmatrix} y_{Aa(1V)} & 0 & 0 \\ 0 & y_{Bb(1V)} & 0 \\ 0 & 0 & y_{Cc(1V)} \end{bmatrix} = \begin{bmatrix} 1149425.2873 - j2873563.2184 & 0 & 0 \\ 0 & 1149425.2873 - j2873563.2184 & 0 \\ 0 & 0 & 1149425.2873 - j2873563.2184 \end{bmatrix}$$

Calculate $\mathbf{Y}_{prim} = \mathbf{A}\mathbf{N}\mathbf{B}\mathbf{Y}_{sc(1V)}\mathbf{B}^T\mathbf{N}^T\mathbf{A}^T =$

$$\begin{bmatrix} 0.00017383 - j0.00043457 & -8.69131e-05 + j0.00021728 & -8.69131e-05 + j0.00021728 & -0.00480529 + j0.01201322 & 0.00240264 - j0.00600661 & 0.00240264 - j0.00600661 \\ -8.69131e-05 + j0.00021728 & 0.00017383 - j0.00043457 & -8.69131e-05 + j0.00021728 & 0.00240264 - j0.00600661 & -0.00480529 + j0.01201322 & 0.00240264 - j0.00600661 \\ -8.69131e-05 + j0.00021728 & -8.69131e-05 + j0.00021728 & 0.00017383 - j0.00043457 & 0.00240264 - j0.00600661 & 0.00240264 - j0.00600661 & -0.00480529 + j0.01201322 \\ -0.00480529 + j0.01201322 & 0.00240264 - j0.00600661 & 0.00240264 - j0.00600661 & 0.13283854 - j0.33209634 & -0.06641927 + j0.16604817 & -0.06641927 + j0.16604817 \\ 0.00240264 - j0.00600661 & -0.00480529 + j0.01201322 & 0.00240264 - j0.00600661 & -0.06641927 + j0.16604817 & 0.13283854 - j0.33209634 & -0.06641927 + j0.16604817 \\ 0.00240264 - j0.00600661 & 0.00240264 - j0.00600661 & -0.00480529 + j0.01201322 & -0.06641927 + j0.16604817 & -0.06641927 + j0.16604817 & 0.13283854 - j0.33209634 \end{bmatrix}$$

Exported $\mathbf{Y}_{prim}$ from OpenDSS:

	G						B					
	↓						↓					
1	0.000174	-0.00043	-8.69E-05	0.000217	-8.69E-05	0.000217	-0.00481	0.012013	0.002403	-0.00601	0.002403	-0.00601
2	-8.69E-05	0.000217	0.000174	-0.00043	-8.69E-05	0.000217	0.002403	-0.00601	-0.00481	0.012013	0.002403	-0.00601
3	-8.69E-05	0.000217	-8.69E-05	0.000217	0.000174	-0.00043	0.002403	-0.00601	0.002403	-0.00601	-0.00481	0.012013
4	-0.00481	0.012013	0.002403	-0.00601	0.002403	-0.00601	0.132839	-0.3321	-0.06642	0.166048	-0.06642	0.166048
5	0.002403	-0.00601	-0.00481	0.012013	0.002403	-0.00601	-0.06642	0.166048	0.132839	-0.3321	-0.06642	0.166048
6	0.002403	-0.00601	0.002403	-0.00601	-0.00481	0.012013	-0.06642	0.166048	-0.06642	0.166048	0.132839	-0.3321
	1		2		3		4		5		6	

Thank you!